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First Grant - Revised 2009 PROPOSAL

Document Status: With Submitter

EPSRC Reference:

Applied Mathematics

Organisation where the Grant would be held

Organisation	University of Portsmouth	Research Organisation Reference:	First Grant Proposal
Division or Department	Faculty of Technology		

Project Title [up to 150 chars]

Control-based bifurcation analysis for experiments

Start Date and Duration

a. Proposed start date	23 April 2012	b. Duration of the grant (months)	24

Applicants

Role	Name	Organisation	Division or Department	How many hours a week will the investigator work on the project?
Principal Investigator	Dr Jan Sieber	University of Portsmouth	Faculty of Technology	18.75

Objectives

List the main objectives of the proposed research in order of priority [up to 4000 chars]

Objective 1: develop methods that permit tracking of bifurcations and singularly perturbed periodic orbits in experiments with feedback control

Objective 2: test the developed algorithms using prototype mechanical experiments set up at the Technical University Denmark (DTU Lyngby)

Summary

Describe the proposed research in simple terms in a way that could be publicised to a general audience [up to 4000 chars]. Note that this summary will be automatically published on EPSRC's website in the event that a grant is awarded.

Many phenomena that are predicted to exist by mathematical theory remain invisible in real life. Yet, mathematical theory also predicts that these hidden phenomena determine our fate when real life is "on the edge". For example, a small increase of wind strength can abruptly cause a bridge cable to start swinging violently. Still more puzzling, the bridge cable may continue to swing strongly even if the wind strength decreases again. Mathematical theory reveals that the mechanisms behind these striking sudden changes (often catastrophes from the point of view of engineering) are universal: they apply to a bridge cable as well as to an ocean current or a neuron. The observed change is abrupt only because the missing link between the two different visible behaviours is typically a phenomenon that is unstable or too sensitive to be visible. This insight enables engineers and scientists to predict, and avoid or control, sudden changes whenever they can rely on a set of equations describing the motion.

This research will develop a method, "control-based continuation", that enables experimenters to observe unstable phenomena directly in controlled laboratory experiments. Control-based continuation uses control to convert the relation between experimental inputs and outputs into an equation that can be solved computationally. Every phenomenon that is natural in the uncontrolled experiment can be found as a solution of this equation. Mechanical prototype experiments (using, for example, pendula and beam-magnet arrangements) have shown that the method is indeed feasible. This project aims to make control-based continuation applicable to more complex experiments and more complex phenomena.

The PI will collaborate with experimenters at the Technical University of Denmark (Lyngby) who investigate vibrations in fast rotating machinery.

One specific objective of the project is to develop and test the continuation of the exact boundaries between stability and instability (so-called bifurcations). Traditional computational methods for determining bifurcations are not applicable to equations extracted from measurements because they rely on the ability to solve the equation

with high accuracy (8-16 significant digits), which is not achievable in most experiments.

Academic Beneficiaries

Describe who will benefit from the research [up to 4000 chars].

(1) Researchers in dynamical systems and their applications

The proposed research will extend the applicability of traditional dynamical systems methods from the purely computational domain to the controlled laboratory environment. This will open new avenues of research in those areas where bifurcation analysis has been successfully applied to models. The techniques developed in this project will enable similar analysis now directly in experiments, permitting validation of the models in regions that are inaccessible in conventional experiments.

(2) Researchers in theoretical and applied neuroscience

(3) Researchers investigating the effects of nonlinearities in mechanical and electrical engineering.

Both communities have traditionally pioneered the application of dynamical systems methods in computer simulations. Current activities (various groups planning to set up experiments) indicate strong initial interest in applying control-based continuation in laboratory experiments, too. Again, their medium-term motivation is validation of model predictions. In the long run these experiments may also lead to discoveries of new effects when the experimental results deviate from model predictions in the newly accessible regimes.

Impact Summary

Impact Summary (please refer to the help for guidance on what to consider when completing this section) [up to 4000 chars]

The proposed research develops a new experimental technique that permits experimenters to observe phenomena that are inaccessible in conventional experiments. The project transfers dynamical systems methods that have been very successful in computer simulations to a new environment: experimental laboratories.

The most immediate impact is expected through uptake of the developed methods by experimenters in engineering (mechanical and electrical) and neuroscience, initially in academic laboratories. The PI will advance this uptake through direct collaboration with experimenters in Bristol and Lyngby (Denmark) and is currently exploring the potential for collaboration with other experimental groups (visits are planned).

Two directions in which the proposed research will find its way into applications are currently on the horizon.

(1) High-tech companies require extensive testing of new machinery and

product components. This method extends the range of feasible laboratory testing conditions.

(2) Neuroscience researches new treatments for conditions such as epileptic seizures, Parkinson's disease or migraine. New insights about the dynamics of neurons will help inform this research. Furthermore, the proposed methods are a new robust way to make feedback control non-invasive (that is, applying control without changing the natural behaviour). Non-invasive control is one of the potential treatments currently discussed in neuroscience. Thus, the project may contribute to neuroscience by providing a new avenue for interaction with the neurosystem without disturbing its natural state.

Summary of Resources Required for Project

Financial resources

Summary fund heading	Fund heading	Full economic Cost	EPSRC contribution	% EPSRC contribution
Directly Incurred	Staff	0.00	0.00	80
	Travel & Subsistence	8000.00	6400.00	80
	Other Costs	910.00	728.00	80
	Sub-total	**8910.00**	**7128.00**	
Directly Allocated	Investigators	53855.00	43084.00	80
	Estates Costs	5568.00	4454.40	80
	Other Directly Allocated	0.00	0.00	80
	Sub-total	**59423.00**	**47538.40**	
Indirect Costs	Indirect Costs	38402.00	30721.60	80
	Total	**106735.00**	**85388.00**	

Summary of staff effort requested

	Months
Investigator	12
Researcher	0
Technician	0
Other	0
Visiting Researcher	0
Student	0
Total	12

Other Support

Details of support sought or received from any other source for this or other research in the same field.
Other support is not relevant to this application.

Staff

Applicants

Role	Name	Post will outlast project (Y/N)	Contracted working week as a % of full time work	Total number of hours to be **charged** to the grant over the duration of the grant	Average number of hours per week **charged** to the grant	Rate of Salary pool/banding	Cost estimate
Principal Investigator	Dr Jan Sieber	Y	100	1650	18.8	53855	53855
						Total	53855

Pathways to impact statement

Summary The proposed research will ultimately achieve its impact through the uptake of the developed methodology by experimenters, especially in engineering and neuroscience. The primary route toward this uptake are successful demonstrations of feasibility by leading experimenters. To push along this route, the PI will arrange a series of research visits to several experimental groups – those mentioned in the proposal, as well as other groups (mentioned below) that have indicated a strong interest in the results. Overall, the PI plans to spend 20% of his time to support the set-up of concrete experiments. The results will be dissiminated in leading academic journals and at the relevant conferences in applied mathematics as well as the respective experimental fields.

Rationale The proposed research develops a new experimental technique that permits experimenters to make observations which are inaccessible in conventional experiments. It transfers methods that have traditionally been usable only in computer simulations to the environment of controllable laboratory experiments. At its heart this is fundamental mathematical research enabling researchers in other fields to extract new information from their experiments.

Hence, the proposed research will find its path toward impact on society only through uptake of the resulting methodology by (at the current stage predominantly academic) experimental resarchers. This requires above all *successful demonstrations in realistic experiments* to convince inter-disciplinary collaborators that the methods suggested in this proposal are viable and help them to answer questions of interest to them. Thus, the PI plans to spend 20% of his allocated time to support concrete plans to set up experiments in several laboratories. Apart from the collaboration with the group at DTU Lyngby (Denmark), which is described in detail in the Case for Support, several more collaborations are still at the planning stage. In the Justification of Resources the PI seeks support to increase the impact of the proposed research through visits to set up further collaborations with experimenters.

The outcomes of the experimental demonstrations, if successful, can then be published in high-impact journals in the fields *relevant to the corresponding experimental community*, which will be in turn a step toward establishing control-based continuation techniques in these communities. The following paragraphs list avenues that have opened up recently, what their their potential impact on society is in the long run, and how the PI plans to support their pursuit.

Neurophysiology The main motivation for neuroscientists to study the internal dynamics and control of neurons (in isolation, in networks or slices) is the development of clinical treatments for conditions such as Parkinson's disease or epileptic seizures. Symptoms of these conditions are understood to be abnormal synchronisation, which is an instability of the equilibrium at the macroscopic level [2, 3]. Suppressing this instability without suppressing natural neuronal activity is a long-term goal of ongoing research, and non-invasive feedback control is one of the perspective approaches. The PI's main collaborative contacts in this area (apart from KT Tsaneva-Atanasova, Bristol) will be S. Rodrigues (SR, Bristol, on move to Univ. Granada, Spain) and M. Desroches (MD, Bristol, on move to INRIA, France). SR will set up experiments on electrical circuits mimicking neurons in the Computational Neuroscience Laboratory of Prof. Eduardo Ros (Univ. Granada) during his initial six-months stay. This is a preparatory step for dynamic clamp experiments planned by SR and MD in collaboration with L. Milescu (Univ. Missouri). Both, SR and MD are keen to put the methods developed during the proposed research to the test and to profit from the PI's expertise acquired in previous experiments and their analysis. In the Justification of Resources money for visits to both laboratories (Ros at Granada and Milescu at Missouri) and to INRIA is requested. All collaborators (KT, SR and MD) and their experimental contacts see potential to make control-based continuation a widely used technique in experimental neuroscience.

Pathways to Impact

Mechanical Engineering A strong driver for experimental innovation are high-tech companies in the aerospace and automotive industries. One of the central research themes of the Dynamics and Control Research Group at the University of Bristol is *hybrid testing*, linking physical experiments and computer simulations bidirectionally and in real time (DJ Wagg and SA Neild are the PI's primary collaborators on the experimental side in this group). The driving force behind these new techniques are the requirements for extensive testing that aerospace companies have to meet. Control-based continuation extends the range of feasible testing regimes. In particular, it removes causality problems in hybrid test problems with rigid connection between simulation and physical test specimen [1, 5]. The methods developed as part of the proposed research will permit the direct tracking of dynamical stability boundaries in tests. Primary industrial collaborators in the research of hybrid testing in Bristol are AgustaWestland (a helicopter manufacturer) and the Airbus Landing Gear Group. Money for visits to Bristol will not be requested but the PI has established a long history of close collaboration with the University of Bristol and plans to continue regular visits (the PI has the status of a visiting Research Fellow at the University of Bristol).

Similarly, the motivation for bifurcation analysis of fast rotating machinery at the DTU are problems posed by the DTU's industrial contacts (specifically, Toyota for the fast rotating machinery [4]). The DTU group explores innovative techniques using prototype experiments and computer simulations, which then get evaluated for commercial adoption. Again, control-based continuation has the potential to extend the range of conditions under which machinery can be tested. Collaboration with the DTU is central to the proposal and will help the PI to adapt his research to the needs of experiments with realistic complexity. Money for two visits to the DTU is sought in the Justification of Resources.

Knowledge transfer between communities All of the above experimental programmes require a significant amount of research on issues specific to their environment such as the control design for the particular experiment and the interface between the numerical methods and the real-time control environment. Several of the collaborators mentioned above have already devoted substantial resources to this avenue of research but have desired collaboration with the PI. Thus, even though the PI does not plan to do the bulk of specific research for all of the above cases, he will help to transmit transferable know-how between the collaborators. In particular, expertise at INRIA in control (A. Rapaport, Montpellier, and the SISYPHE group) and expertise at DTU in numerical continuation techniques (F. Schilder) promise to add value to the basic techniques developed in the proposed research across disciplines.

References

[1] P.J. Gawthrop, S.A. Neild, A. Gonzalez-Buelga, and D.J. Wagg. Causality in real-time dynamic substructure testing. *Mechatronics*, 19(7):1105 – 1115, 2009. Special Issue on Hardware-in-the-loop simulation.

[2] F. Marten, S. Rodrigues, P. Suffczynski, M. P. Richardson, and J. R. Terry. Derivation and analysis of an ordinary differential equation mean-field model for studying clinically recorded epilepsy dynamics. *Phys. Rev. E*, 79(2):021911, Feb 2009.

[3] O. V. Popovych and P. A. Tass. Synchronization control of interacting oscillatory ensembles by mixed nonlinear delayed feedback. *Phys. Rev. E*, 82(2):026204, Aug 2010.

[4] F. Schilder, J. Rübel, J. Starke, H. Osinga, B. Krauskopf, and M. Inagaki. Efficient computation of quasiperiodic oscillations in nonlinear systems with fast rotating parts. *Nonlinear Dynamics*, 51:529–539, 2008.

[5] J. Sieber and B. Krauskopf. Tracking oscillations in the presence of delay-induced essential instability. *Journal of Sound and Vibrations*, 315(3):781–795, 2008.

Pathways to Impact

Control-based bifurcation analysis for experiments

Case for Support — Jan Sieber

The proposed research develops a new method that permits experimenters to observe phenomena which are inaccessible in conventional experiments due to their dynamical instability or sensitivity to disturbances. The proposed method does not require an accurate mathematical model, making it particularly attractive for experiments with nonlinearities and complex behaviour in engineering and the biological sciences. The proposal combines ideas from feedback control, nonlinear dynamical systems, and numerical analysis.

1 Previous research and track record

Dr Jan Sieber was appointed as a Senior Lecturer in the Department of Mathematics at the University of Portsmouth in 2008. His research expertise is in the area of dynamical systems. The main subject of his research are dynamical systems with focus on semiconductor laser design, dynamical systems with delay or switching, and the development of numerical continuation techniques for physical experiments.

The PI's research includes both the development of general mathematical theory, and the development and verification of new experimental methods. His publication record is proof of the PI's ability to collaborate with experimenters in science and engineering. In several interdiscpinary projects the PIs results and active engagement have driven the projects and opened new avenues. Currently, the PI serves as an Associate Editor on the Editorial Board of the SIAM Journal on Applied Dynamical Systems and is co-editing (with JMT Thompson, FRS) an issue of the Philosophical Transactions of the Royal Society (Series A) on the influence of nonlinearity and randomness on climate prediction. Regular invitations to (invitation-only) workshops at the Mathematical Biosciences Institute (Univ. Ohio, US, 2011), at the IUTAM symposium in Aberdeen (2010), and at the Banff International Research Station (Alberta, Canada, 2009) show that the PI's research has gained recognition in the communities researching dynamical systems and their applications to bio-sciences and engineering. The paragraphs below highlight scientific results to which the PI has made a significant contribution.

Research track record — Semiconductor laser design The paper [S11] predicted a range of delayed feedback effects that are directly relevant to applications in optical telecommunication: high-frequency oscillations, excitability and chaos [S9]. These predictions led to the design of a new laser, introduced in [S10], which displays all of these effects in a finely controllable manner, and which continues to draw scientific interest (see [16] for a review).

Dynamical systems with delay or switching A theoretical result by the PI in [S7] shows that it is possible to stabilise unstable equilibria with feedback control for arbitrarily large delays if the control input consists of a simple switch (relay). The paper [S7] also develops a local bifurcation theory for systems with delayed switching, discovering several surprising features, for example, a transition to small-scale chaos by the introduction of an arbitrarily small delay into a switch. A direct application of the PI's research on delay to mechanical experiments was a numerical stability result for delay compensation schemes employed in hybrid testing [S8]. In two recent papers the PI proved that characteristic matrices exist for periodic linear delay differential equations (DDEs) [S2], similar to the characteristic matrices for linear DDEs with constant coefficients, and that periodic orbits in a DDE with state-dependent delays are given as roots of a system of smooth algebraic equations [S1]. The paper [S1] is remarkable in that it removes long-standing functional-analytic difficulties caused by state-dependent delays for periodic problems [7].

Control-based continuation for experiments The idea for this new experimental method started with the observation in [S6] that the time-delayed feedback methods popular in the nonlinear dynamics community in physics [10, 15] are equivalent to a simple relaxed fixed point iteration of a nonlinear fixed point problem. This led to the proposal for a general method for tracking unstable phenomena in experiments in [S5] and to the first demonstration using a parametrically excited mechanical pendulum [S4]; this result was featured in Physical Review Focus (focus.aps.org/story/v22/st1). Barton *et al.* [1, 2] and Bureau *et al.* [3] have implemented the method introduced in [S5] and, hence, independently verified the viability of control-based continuation. This new experimental method proposed by the PI has already attracted substantial interest in engineering and neuroscience. Two currently funded EPSRC grants expressly aim to apply this method to hybrid testing and bridge cables (EP/F030711/1, PI Prof. DJ Wagg), and clamped neurons (EP/I018638/1 by PI Dr KT Tsaneva-Atanasova).

The ideas developed in [S4, S5], accompanied with new theoretical insights from [S1], form the background for the research in the proposed project.

The University of Portsmouth The Department of Mathematics has recently formed a strong research group working on theory and applications of dynamical systems consisting of seven academics (five of those, including the PI, were hired 2008 or later). The department was part of the Applied Mathematics submission to the RAE 2008, which gained grades of 4 (15%), 3 (60%) and 2 (25%). The project will benefit from the expertise of several members of the group, in particular, Thomas Waters (expertise in continuation problems and orbit design), Andrew Burbanks (numerical methods for dynamical systems) and Murad Banaji (topological methods).

The University's support for the PI's research The University has agreed to fund one position for a research assistant at the postdoctoral level specifically to support the PI's research proposed for this project and the PI's collaboration with Thomas Waters. The Department has advertised for a postdoctoral researcher with the necessary expertise to work on the development of continuation methods designed for ill-conditioned problems. This will greatly assist the PIs work on method development (see Programme). The research assistant will be expected to start in autumn 2011 and will be employed for the duration of two years.

Selected Publications cited as [S1–S11], see References for other citations). The PI has contributed 50% or more to publications marked with asterisk. Publications marked with † are papers presenting *experimental results* where the PI either contributed to the analysis or proposed the experimental methodology.

∗ [S1] J. Sieber. Dimension reduction for periodic boundary value problems of functional differential equations. submitted, http://arxiv.org/abs/1010.2391, 2011.

∗ [S2] J. Sieber and R. Szalai. Characteristic matrices for linear periodic delay differential equations. *SIAM J. Appl. Dyn. Sys.*, 10(1):129–147, 2011.

† ∗ [S3] J. Sieber, B. Krauskopf, D. J. Wagg, S. Neild, and A. Gonzalez-Buelga. Control-based continuation of unstable periodic orbits. *ASME Journal of Computational and Nonlinear Dynamics*, 6(1):011005, 2011.

† ∗ [S4] J. Sieber, A. Gonzalez-Buelga, S.A. Neild, D.J. Wagg, and B. Krauskopf. Experimental continuation of periodic orbits through a fold. *Phys. Rev. Lett.*, 100(244101), 2008. Featured in Physical Review Focus (focus.aps.org/story/v22/st1)

∗ [S5] J. Sieber and B. Krauskopf. Control based bifurcation analysis for experiments. *Nonlinear Dynamics*, 51(3):365–377, 2008.

∗ [S6] J. Sieber and B. Krauskopf. Control-based continuation of periodic orbits with a time-delayed difference scheme. *Int. J. of Bifurcation and Chaos*, 17(8):2579 – 2593, 2007.

∗ [S7] J. Sieber. Dynamics of delayed relay systems. *Nonlinearity*, 19(11):2489–2527, 2006.

† [S8] M.I. Wallace, J. Sieber, S.A. Neild, D.J. Wagg, and B. Krauskopf. Stability analysis of real-time dynamic substructuring using delay differential equation models. *Int. J. of Earthquake Engineering and Structural Dynamics*, 34(15):1817–1832, 2005.

† [S9] S. Bauer, O. Brox, J. Kreissl, B. Sartorius, M. Radziunas, J. Sieber, H.-J. Wünsche, and F. Henneberger. Nonlinear dynamics of semiconductor lasers with active optical feedback. *Phys. Rev. E*, 69(016206), 2004.

† [S10] S. Bauer, O. Brox, J. Sieber, and M. Wolfrum. Novel concept for a tunable optical microwave source. In *Proceedings of the OFC, Optical Society of America*, 2002.

∗ [S11] J. Sieber. Numerical bifurcation analysis for multi-section semiconductor lasers. *SIAM J. of Appl. Dyn. Sys.*, 1(2):248–270, 2002.

2 Description of the proposed research

2.1 Rationale

The focus of the proposed research is the direct continuation of stability boundaries (*bifurcations*) in complex and realistic systems such as fast rotating machinery and spiking neurons. The methods developed under this project will benefit experimenters in science and engineering because they enable them to investigate phenomena in experiments that have so far evaded observation due to their dynamical instability or extreme sensitivity to perturbations. This, in turn, will make new experimental observations possible, and validate and correct associated models in regimes that are as yet inaccessible in conventional experiments. The overriding criterion of the proposed methods is *feasibility in an experiment*, which is fundamentally different from the well-researched numerical efficiency. Hence, this criterion opens new theoretical questions and requires the development of new algorithms.

Successful prototype experiments on mechanical oscillators have laid the foundation of the proposed work, which is all about applying continuation techniques to experiments with more complex and realistic structures (this proposal will focus on neurons and fast rotating machinery), and more complex phenomena such as bifurcations and periodic orbits in singularly perturbed systems.

2.2 Background

Recent work Recent experimental results by Sieber *et al.* [S3,S4] and Barton *et al.* [1, 2] have demonstrated that it is possible to find and track unknown dynamically unstable phenomena in physical experiments accurately. The experiments were mechanical oscillators that showed dynamically unstable periodic motion. Notably, the existence of the unstable periodic motion was not inferred indirectly but measured directly as an output of the experiment. For example, movies of the unstable rotation of a forced pendulum are shown in Physical Review Focus (`http://focus.aps.org/story/v22/st1#videos`).

Background on methods While the method employed in these experiments, control-based continuation, started out as a modification of Pyragas' time-delayed feedback control [10, 15], the proposal by Rulkov *et al.* [11] is the closest predecessor in terms of methodology.

Figure 1 outlines the method for the driven pendulum [S3, S4]. One wants to learn about (possibly unstable) periodic orbits of the *uncontrolled* experiment (the box with the pendulum and only periodic excitation as input). In order to do so, one adds a feedback control loop to the setup (symbolised by the arrows connecting output, "−", demand and controller back to the "+").

If the feedback control loop is linearly stabilising, the output y (after transients have settled) depends locally uniquely on the inputs, which are in the example the excitation parameters $\mu = (p, \omega)$, and the periodic demand signal y_d. This unique dependence defines a nonlinear map $Y : (y_d, \mu) \mapsto y$ in the space of periodic functions. Periodic orbits of the uncontrolled system can then be found as solutions of the nonlinear fixed point equation

$$y_d = Y(y_d, \mu). \qquad \text{(Eq. 1)}$$

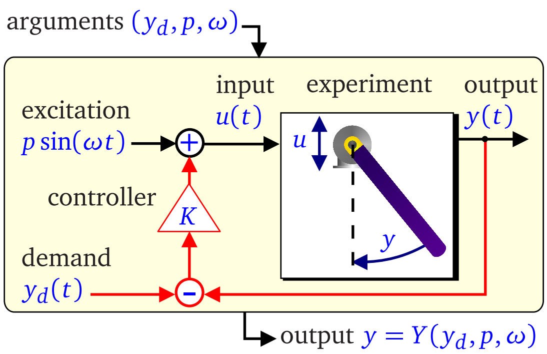

Figure 1: *Working principle of control-based continuation, originally proposed by Sieber & Krauskopf* [S5].

This at first sight cumbersome approach has the advantage that it finds periodic orbits regardless of their stability in the original system. In principle, the approach has been generalised to equilibria and periodic orbits of autonomous systems (where the period is unknown) [S5], however, no experiments were performed for the set-up proposed in [S5]. The accuracy of the result, that is, the difference between the observed output y of the experiment with feedback control and the hypothesised unstable orbit of the original (uncontrolled) experiment, is only limited by the accuracy of the measurement instruments. In particular, the accuracy of the result does not depend on the accuracy of a model. One can then adapt numerical methods such as Newton iterations and pseudo-arclength continuation [5] to find and track solutions of the fixed point problem (Eq. 1). This proposal addresses the difficulties that numerical methods are facing if they are applied to equations extracted from experiments, especially to (Eq. 1).

Ongoing work on experimental control design A central point in Fig. 1 is the presence of the linearly stabilising feedback loop. The design of this feedback loop depends strongly on the particular experiment. In general one needs at least a crude model of the experimental dynamics to design the feedback control (in the worst case obtained by system identification). For the demonstrations in [S3, S4] and [1, 2] simple PD (proportional-plus-derivative) control was used. For more complex experiments the design of the feedback loop is a challenge that has to be mastered before continuation can be attempted. Two experimental groups are currently receiving funding from the EPSRC with the express aim to enable continuation in experiments based on the diagram Fig. 1: the group of D.J. Wagg (Mech. Eng., Univ. Bristol) investigates mechanical experiments in the context of *hybrid testing* (or real-time dynamic substructuring) in civil and mechanical engineering (EP/F030711/1).

K. Tsaneva-Atanasova investigates practical controllability of clamped neurons in bursting regime with the mid-term aim to apply continuation to clamped neurons in an in-vitro environment (EP/I018638/1).

A collaboration between the group of J. Starke (Mathematics) and I. Santos & J.J. Thomsen (Mechanical Engineering) at the DTU Lyngby (Denmark) is currently setting up infrastructure for the experimental bifurcation analysis of machine vibrations based on control-based continuation (see [3] for first experimental results).

The methods developed as part of the proposed research will be of direct benefit to the above ongoing projects by widening their scope and making bifurcations and singularly perturbed phenomena directly accessible in their experiments.

2.3 Objectives and problem statement

Objectives This project's aim is to make control-based continuation applicable for continuation of bifurcations and for neuroscience laboratory experiments. Specifically, the aim is to develop the necessary methods and background theory for the *outside* of the box containing the feedback loop in diagram Fig. 1. This goal will be pursued in two strands

- **(M)** Development of methods specific for the continuation of bifurcations and singularly perturbed periodic orbits under conditions of low accuracy output.
- **(L)** Testing of algorithms developed in strand (M) using prototype experiments set up at DTU and model-based studies of more complex experiments.

In addition, strand (L) will also link methods as developed and tested in (M) into the general-purpose platform COCO [4] currently developed and maintained at the DTU. The PI plans to spend two extended research visits at the DTU where he will profit from the experimental infrastructure and extensive collaboration with experts in numerical analysis (F. Schilder, the author of COCO) and control and mechanical engineering (E. Bureau, the lead author of the feasibility study [3]).

Problems to be tackled One encounters a specific set of problems when tracking solutions of the nonlinear fixed point problem (Eq. 1) (where μ may be a higher-dimensional parameter in the general case). These problems make an approach necessary that differs from the classical methods for low-dimensional dynamical systems [5] but also from the currently intensely researched methods for high-dimensional dynamical systems (typical application areas are fluid dynamics and spatially extended systems; state-of-the-art methods are implemented in LOCA [12]).

Nonlinear systems such as (Eq. 1), which are extracted from experiments by means of control, have three characteristic features:

- (**low accuracy**) parts of the right-hand side (namely the map Y) come with low accuracy: measurement accuracy in the feasibility studies [S3, S4] and [1, 2] yielded 2–3 significant digits. There is limited scope for improving the accuracy by averaging over a longer period of the output y but this will realistically achieve at most one additional significant digit.
- (**expensive**) each evaluation of Y takes far more time than a classical function evaluation because one has to wait until the transients in the controlled experiment have settled down (approximately one second in [S3, S4]). In particular, the evaluation of Y cannot be sped up by parallelisation. Moreover, the difference between arguments in successive evaluations of Y has to be small because the feedback loop in Fig. 1 is often only locally stabilising (this was the case in [S3, S4]).
- (**low dimension**) The space dimension of y_d is typically low because it is limited by the number of available inputs into the experiment. In theory, the overall dimension may still be infinite because the tracked object is a periodic orbit but in the example studies these orbits were nearly harmonic such that the number of variables in the discretised version of the fixed point problem (Eq. 1) was 2 in [S3, S4], 3 in [1], and 12 in [3]. (Responses of neurons will often be spiky rather than nearly harmonic, though.)

These features set problem (Eq. 1) apart from the classical high-dimensional nonlinear problems: the low accuracy of the right-hand side makes it more difficult, but the low problem dimension permits approaches that would be infeasible for high-dimensional systems. The inherent restriction on accuracy sets a strict upper limit on the condition for the type of problems one wants to solve. For example, direct continuation of the saddle-node of periodic orbits observed in [1] by continuing the extended problem (used in AUTO [5]) in a two-dimensional system parameter μ turned out to be infeasible under the given experimental conditions (private communication with the lead author of [1]). The uncertainty in Y is amplified by a factor of order $1/\varepsilon$ when one attempts to approximate $\partial_1 Y$ by a finite difference with step ε ($\partial_1 Y$ is part of the right-hand side in the extended system). Approaches side-stepping explicit differentiation of Y, such as continuing two coexisting periodic orbits with a fixed distance ε, lead to a problem condition (in the linearisation) of order $1/\varepsilon$. The same problem occurs when attempting to continue other local bifurcations (Hopf bifurcation, period doubling): the condition of the linearised problem is amplified by $1/\varepsilon$ if one attempts to locate the bifurcation with accuracy ε.

Similarly, if the underlying system is singularly perturbed then this singularity shows up in the condition of the linearised problem. In simple problems the factor is $1/\varepsilon$ where ε is the parameter measuring the time scale separation (see preliminary study [14]).

2.4 Programme and methodology

(M) Method development

The primary difficulty when one attempts to apply control-based continuation to bifurcations or neurons is that the resulting problems are singularly perturbed, leading to ill-conditioning of the nonlinear problem (Eq. 1).

The two guiding principles to overcoming this difficulty are:

1. replace the differentiation by an exploration of a larger neighbourhood and a topological method on the solution manifold of (Eq. 1);
2. extend the continuation by natural and artificial parameters and equations (and thus, the dimension of the solution manifold) to reduce the condition.

The first principle is illustrated in Fig. 2 for the simplest case, a fold. The fold of an implicitly given surface, which one assumes to be computable only with low accuracy (two significant digits), is determined in two steps: first map out the surface in a patch along the fold (fewer than 900 light dots).

The criterion for bounding the patch and for the fold (dark blue) depends in this illustration on the secants of the triangulation. The fold is determined as a path along which a multi-linear form of the secants changes its sign. In short, one has replaced the fold determination by a two-step process: map the surface with a multi-parameter continuation based on a Newton iteration, and then detect sign-changes on the surface.

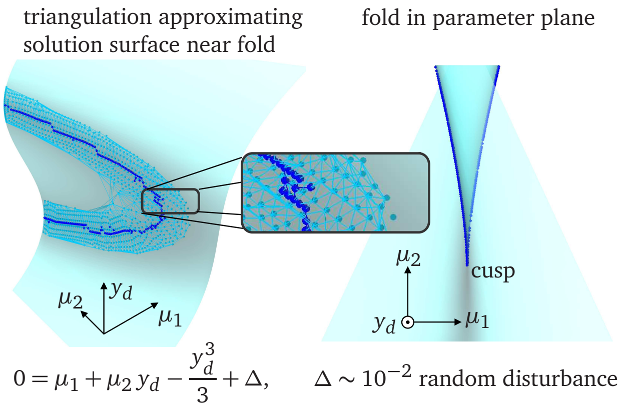

Figure 2: *Demonstration of fold continuation through cusp.*

Combination with the second principle makes this approach also suitable for Hopf and period-doubling bifurcations. Both are (in an appropriate formulation) symmetry-breaking pitchfork bifurcations. After extending the problem with an artificial parameter the pitchfork changes into a cusp. This cusp can then be determined and tracked as a feature of a regular solution manifold of dimension $\dim\mu+1$ (or $\dim\mu+2$ if the period of y_d is unknown) similar to the demonstration in Fig. 2. The corresponding topological method would track points where the desired parameter-level sets cross the boundary of the local neighbourhood of the point four times (indicating a branching of the level set).

The preliminary study [14] demonstrates the second principle for a family of periodic orbits in a singularly perturbed system (a nonlinear oscillator that is close to conservative, $\varepsilon = 10^{-3}$): extending the problem with two additional parameters to have a three-dimensional solution manifold of (Eq. 1) reduces the condition of the nonlinear problem, keeping it bounded uniformly in ε (without extension the condition is $\sim 1/\varepsilon$). In general, the approach is to embed the original ill-conditioned problem into a better-conditioned problem with more parameters. In a second step one has to study how the higher-dimensional manifold, given on a grid with a certain error, intersects a subspace of the parameter space. This intersection problem contains the sensitivity of the original problem (which means that the intersection may be at a tight angle) such that the approach not only gives a solution but also gives a detailed estimate of its sensitivity on system parameters (based on the intersection angle).

The appeal of the above methodology is that the level of sophistication in the underlying numerical and geometric methods can be gradually increased depending on the needs in the experimental part of the programme. Compared to the general-purpose manifold covering algorithms as developed in [8] the algorithms proposed for strand (M) should be significantly simpler because the topology of continued manifold is simple: the manifold is a patch along the bifurcation curve.

(L) Link to experiments and existing platforms

Throughout the project the PI will collaborate with experimental groups at the University of Bristol (DAW Barton and KT Tsaneva-Atanasova, Dept. of Engineering Mathematics) and the Technical University of Denmark, Lyngby (a collaboration between by J Starke, I Santos and JJ Thomsen). While the link to Bristol is well established and research visits have been arranged regularly and on short notice during the previous years, the collaboration with DTU requires more planning and financial support for two extended visits.

One of the central research topics of the DTU group is the analysis of vibrations in fast rotating machinery. During the previous years the group has developed substantial experimental and computational infrastructure, a hierarchy of models, and know-how in control engineering and numerical analysis specific to vibration analysis. Currently the group explores the potential of control-based continuation for their experiments.

(V1) The prototype experiment studied at the DTU [3], a backlash oscillator, is an excellent environment for testing if the simple topological method outlined in Figure 2 will be feasible in experiments. During the first visit (within the first 3 months) the PI will familiarise himself with the hard- and software environment used on the backlash oscillator. The algorithms for the simple fold continuation and for the singularly perturbed (nearly conservative) case [14] will be at a stage at this time that one can start their implementation.

(V2) The bifurcations occurring in the models of fast rotating machinery can be treated as Hopf bifurcations (they are Hopf bifurcations in the limit of high-speed rotation [13]). So, the rotating machinery problems are ideal showcases for the methods in strand (M). Since the implementation of experiments with fast rotating parts and feedback control will exceed the time horizon of the proposal, the algorithms of strand (M) will be tested initially on models from the hierarchy developed at the DTU. The second visit (between months 8 and 10) will be devoted to collaboration on a digitally induced Hopf instability in the backlash oscillator. This permits a first realistic test for the Hopf bifurcation continuation with a finely tunable experimental environment. The combination of simulations and finely tunable experiments will permit the collaborators to gradually adjust the level of difficulty of the problem depending on how successful and robust the methods developed in strand (M) are in this particular environment.

Another benefit of the collaboration with DTU is that the development of the algorithms in strand (M) will profit from the very flexible platform provided by COCO for which F. Schilder is currently adding general-purpose multi-parameter manifold continuation (by incorporating the Multifario covering [8]).

2.5 Academic impact

Three different academic communities will benefit in the short run from progress achieved in this project.

Engineers and mathematicians studying effects of nonlinearity in mechanical and civil engineering will be able to use the methods developed here to validate and correct models. For example, the precise nature and magnitude of damping is notoriously difficult to model in mechanical systems as it often depends on microscopic details of surfaces and the regime in which the system is operating. Damping in the rotating pendulum studied in [S4] is dominated by air drag, whereas it is dominated by impefections of the bearings for small-amplitude oscillations.

The neuroscience community has pioneered the application (and development) of nonlinear methods in the purely computational context [6]. Current activity (project EP/I018638/1, PI KT Tsaneva-Atanasova) shows that control-based continuation is being embraced by experimental neuroscientists, too. As soon as the basic method has been implemented, neuroscience experiments will benefit directly from the results of this project. To increase the potential for impact the PI seeks support to establish other collaborations in this area (see *Pathways to Impact* statement).

A research area that faces similar problems (expensive right-hand sides, evaluated with low accuracy but often of low dimension) is *equation-free coarse-grained* analysis. The coarse-graining methodology has a microscopic simulator (*time stepper*) inside the box in Fig. 1 together with a lifting and restriction procedure that extracts a nonlinear equation, for example, from kinetic Monte-Carlo simulations (see [9] for a recent review). Coarse-graining methods as proposed in [9] are unlikely to be applicable in most physical experiments because they rely on the ability to set the initial conditions of the dynamical system to values specified by the lifting procedure. However, the nonlinear system resulting from coarse graining typically has properties similar to (Eq. 1), in particular, it can often be evaluated with low accurcay only. This means that know-how developed for these methods will be transferable to this project. Vice-versa, results achieved as part of this project will contribute to current efforts to expand the scope of coarse-graining. Coarse-graining analysis of agent-based models is a key area of expertise of the DTU group (J. Starke), which plans to extend the coarse-graining analysis to machine vibrations in parallel to control-based continuation.

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Justification of Resources

Staff – directly allocated

Cost	Item	Justification
50%	**Principal Investigator time**	All of the proposed research will be conducted by the PI (supported by the university-funded RA). The funding will relieve the PI of 67% of his teaching and administrative duties for the duration of 2 years.
	Travel & subsistence	
£2,000	2 Visits to DTU Lyngby (Denmark)	Collaboration central to the proposed resarch; requested funding covers travel & accommodation of PI for 2 × 1 month; collaborators provide access to infrastructure and support in kind (see Case for Support & Letter of Support)
	Conferences	
£1,800	ASME–IDETC (MSNDC) 2013	main international (US) meeting of the engineering community researching on nonlinear dynamics & control; primary platform for initial dissemination to engineering community
£1,300	SIAM Conf. on Dynamical Systems 2013 (Snowbird, US)	main international (US) meeting of the mathematical dynamical systems community; primary platform for initial dissemination to dynamical systems community
	Other directly incurred costs	
£700	2× Individual Research License Simulink for PC & travel laptop	necessary software to prepare algorithms for interfacing to experiments
£210	1× additional research license Matlab for travel laptop	(licensed as part of group license)
	Impact	
£2,900	Visits	Funding is requested for 1-week visits to • *Eduardo Ros* (Univ. Granada, Spain) £600, • *Lorin Milescu* (Univ. Missouri, USA) £1,100, • *Alain Rapaport* (INRIA, Montpellier, France) £600, • SISYPHE group (INRIA, Rocquencourt, France) £600. to explore potential for collaboration and application of proposed research.

Justification for Resources

Diagrammatic Workplan

for EPSRC First Grant proposal *Control-based bifurcation analysis for experiments*

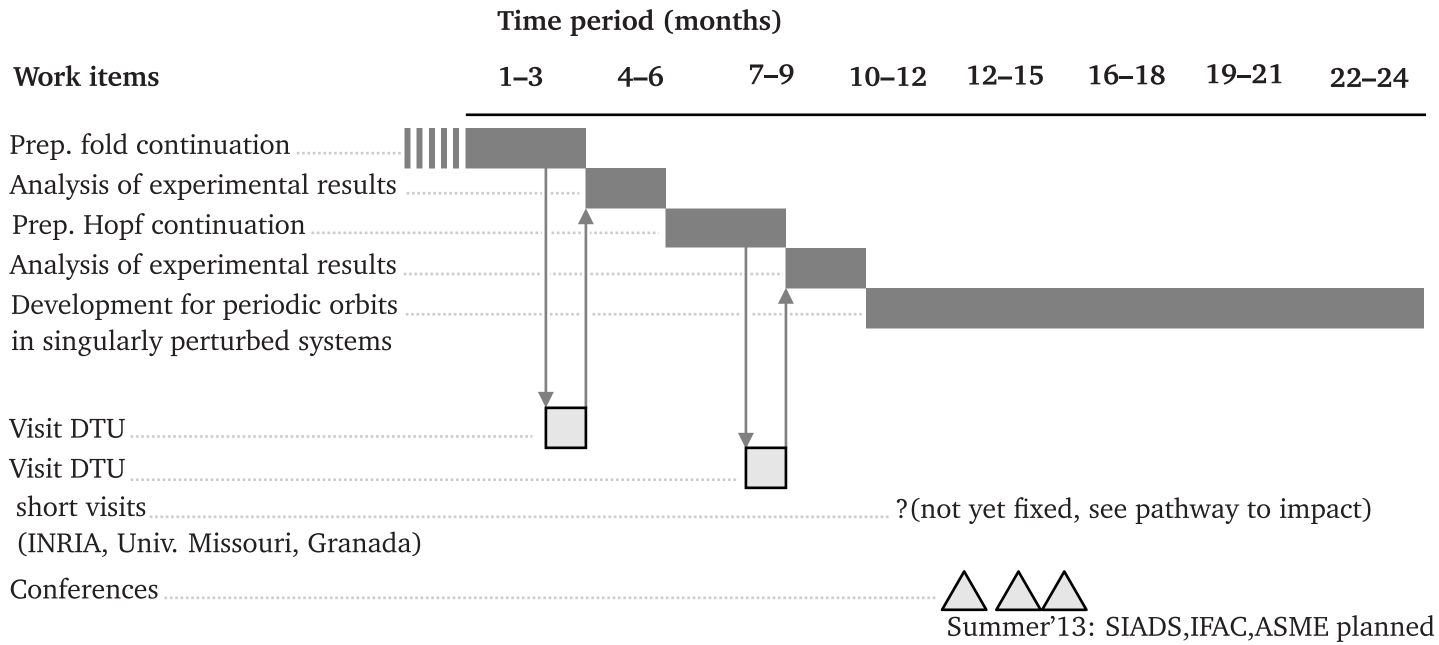

Table 1: *Workplan — See Case for Support and Pathway to Impact statements for details*