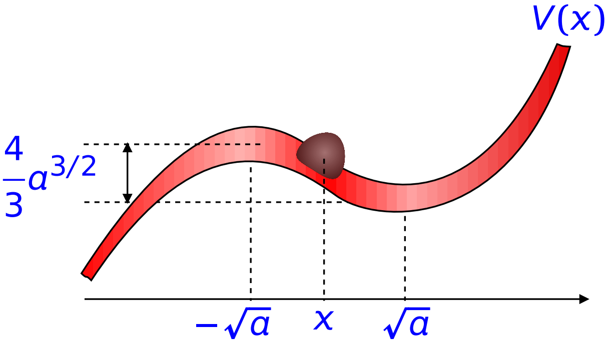
# Outline — 2nd part

- Noise induced escape near tipping
- Estimates of normal form parameters from time series

# Noise induced escape

## overdamped particle in a well

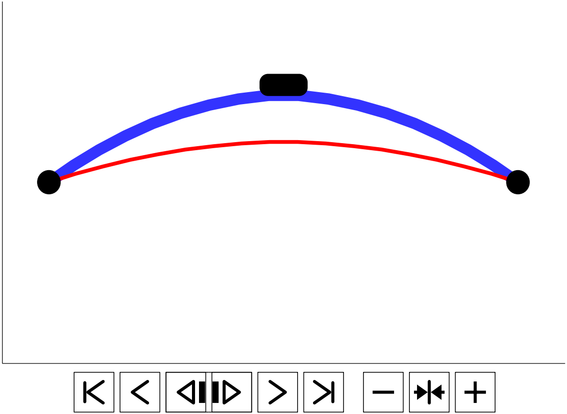



$$\frac{d}{dt}x = -V'(x) = a - x^2 + \textbf{noise}$$

# Mechanical caricature

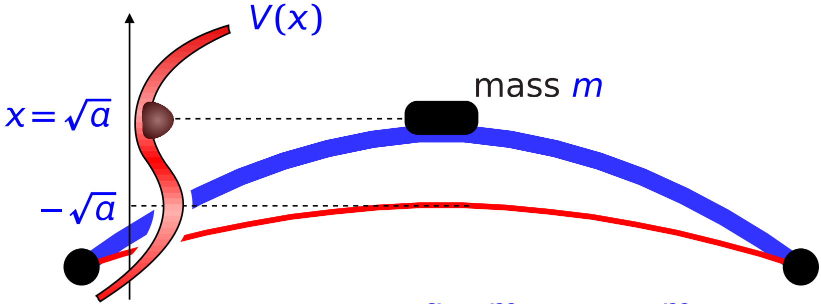
## of positive feedback

- squishy beam, clamped and loaded with gradually increasing mass *m*



# Mechanical caricature

## of positive feedback

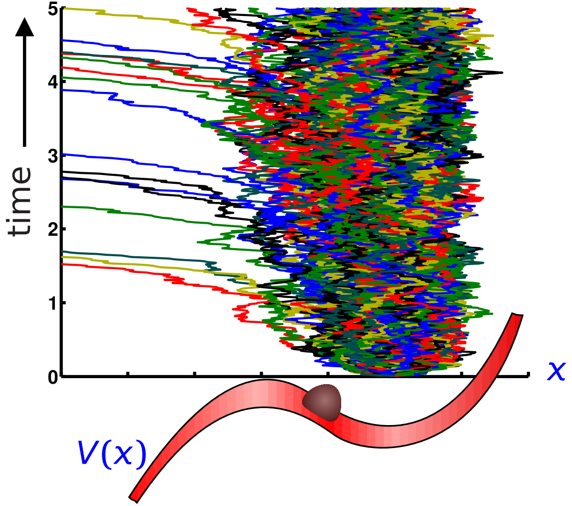



$$a \sim m_{\mathrm{critical}} - m$$

$$\frac{\mathrm{d}}{\mathrm{d}t} x = -V'(x) = a - x^2 \ \textbf{+ noise}$$

# Noise induced escape

## Fixed well depth, Noise amplitude $\sigma > 0$

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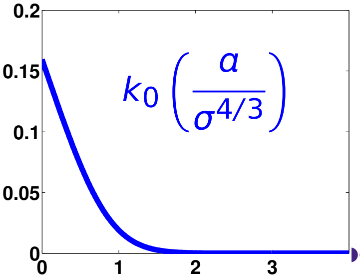
# Noise induced escape

## Fixed well depth, Noise amplitude $\sigma > 0$

- $a \gg \sigma$ (barrier $\gg$ noise amplitude):
  **Kramers' escape rate**

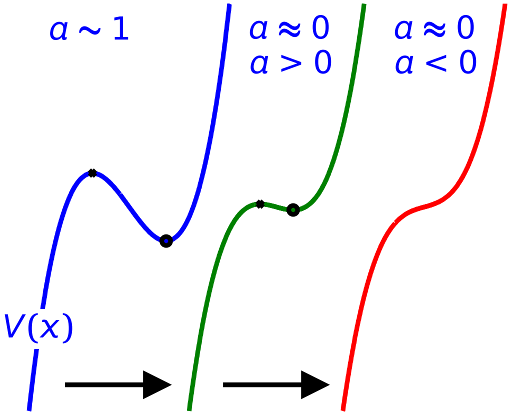
$$k_0\left(\frac{a}{\sigma^{4/3}}\right) \sim \frac{2}{\pi}\sqrt{\frac{a}{\sigma^{4/3}}}\exp\left(-2\sqrt{\frac{a}{\sigma^{4/3}}}\right)$$

- $a \sim \sigma, \quad a \leq \sigma$
  linear boundary value problem for probability density $p(x)$
  **Fokker-Planck**



# Noise induced escape

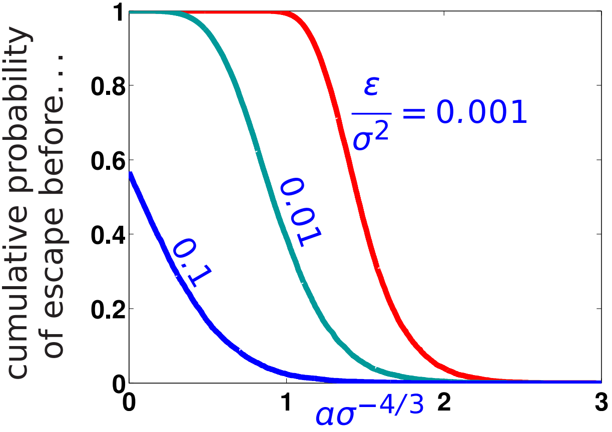
**Shrinking well:** $\frac{\mathrm{d}}{\mathrm{d}t}a = -\varepsilon$, **Noise amplitude** $\sigma > 0$



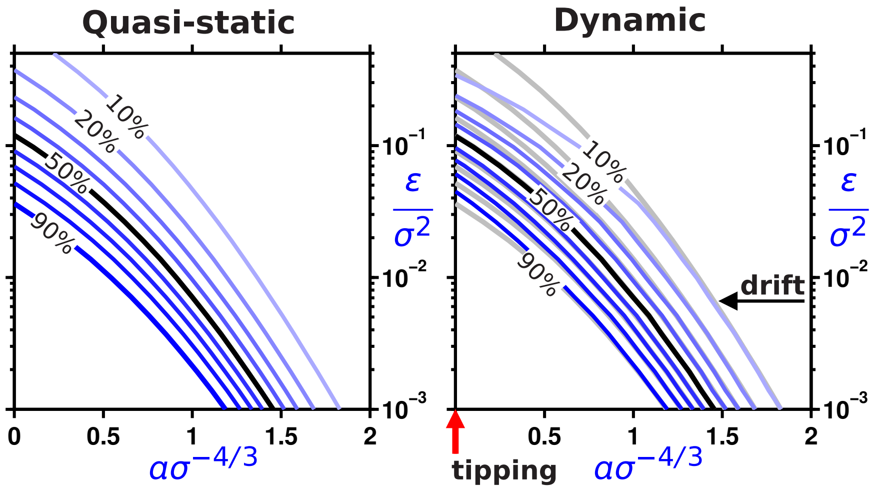
# Noise induced escape

**Shrinking well:** $\frac{\mathrm{d}}{\mathrm{d}t}a = -\varepsilon$, **Noise amplitude** $\sigma > 0$

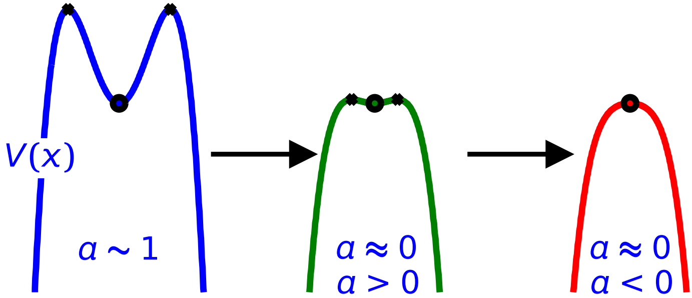
start from $a_0 \gg \sigma^{4/3}$



# Cumulative escape probabilities



# Related work & References

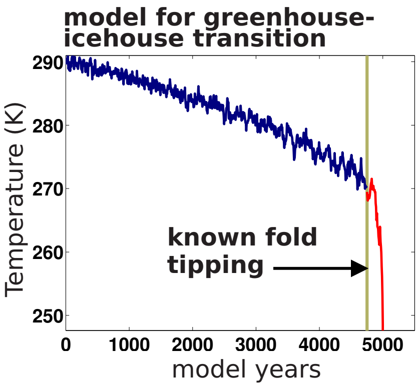


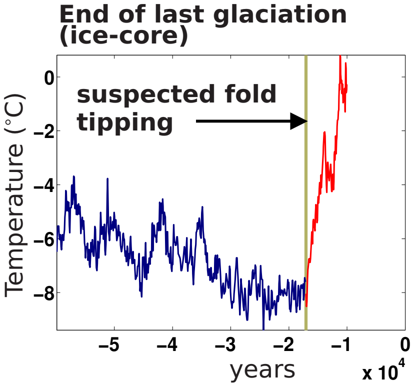

**Early or delayed escape, under-damped**

| | | |
|---|---|---|
| Miller/Shaw | $\Longleftarrow$ | Engineering |
| Berglund/Gentz | $\Longleftarrow$ | Mathematical theory |
| Kuske | $\Longleftarrow$ | Fokker-Plank equations |

# Estimate from time series

## 2 Examples (Dakos *et al*)

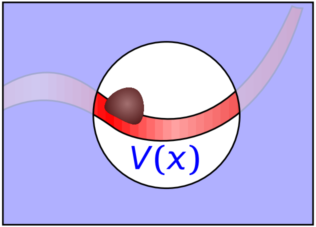



## Estimation procedures:

- Autocorrelation coefficients (Held/Kleinen)
- Detrended fluctuation analysis (Livina/Lenton)

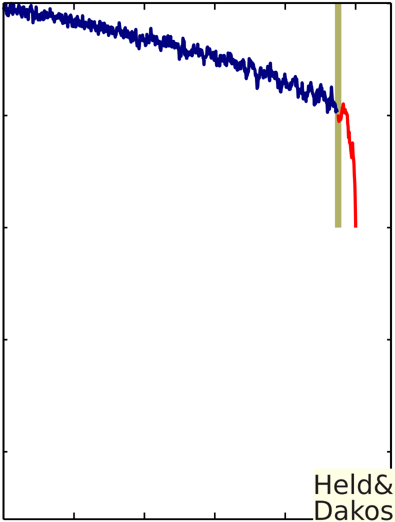
# Estimate from time series




measurements noise

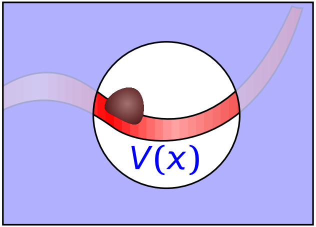
$$x_{n+1} = \mathbf{c}\, x_n + \sigma\, \eta_n$$

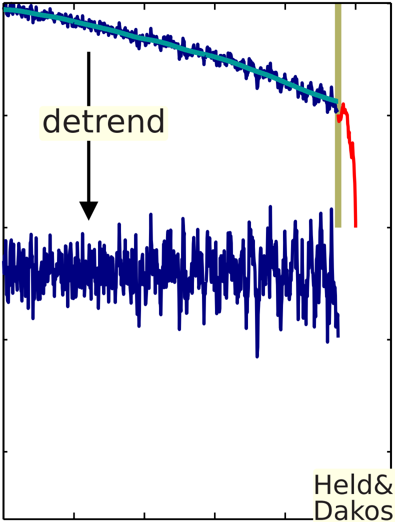
ARC(1)



Held&Kleinen
Dakos *et al*

# Estimate from time series



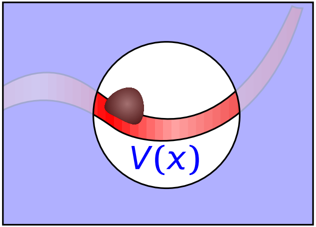

measurements     noise

$$x_{n+1} = \mathbf{c}\, x_n + \sigma\, \eta_n$$

ARC(1)

Held&Kleinen
Dakos *et al*

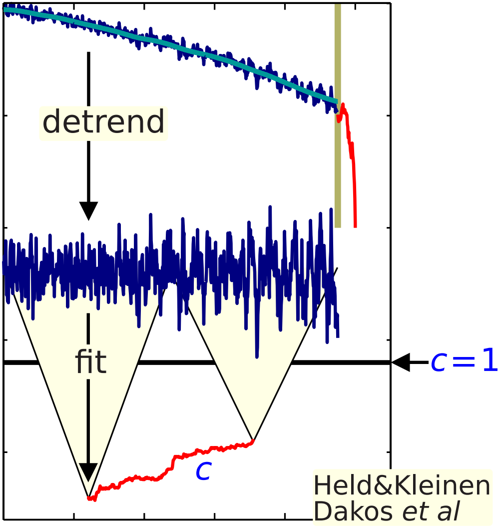
# Estimate from time series




measurements noise

$$x_{n+1} = \mathbf{c}\, x_n + \sigma\, \eta_n$$

ARC(1)

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# Estimate from time series

**Relation of estimates from AR(1) model to normal form quantities**

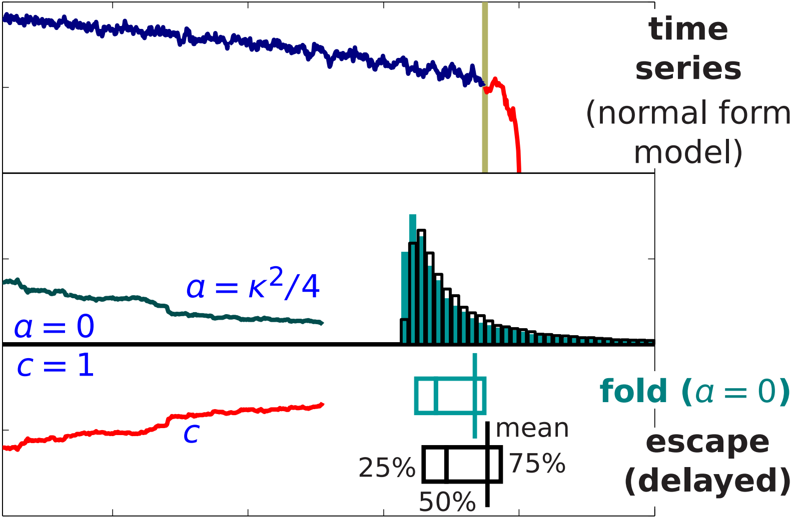
$$x_{n+1} = \mathbf{c} x_n + \sigma \eta_n$$

time step of linearized stochastic process

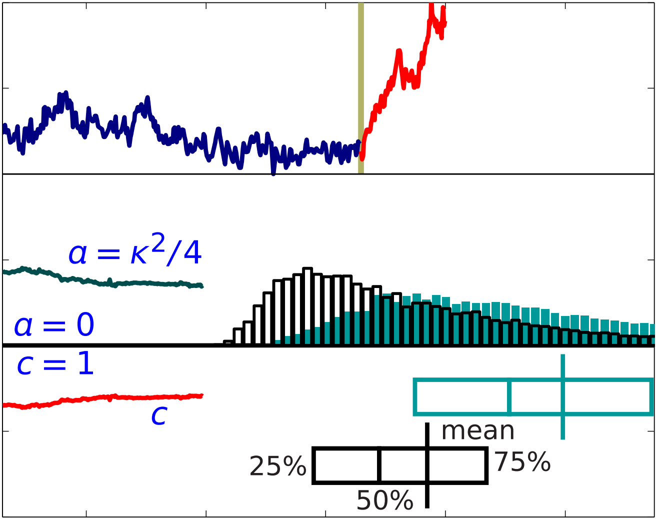
$$x_{n+1} = (\mathbf{1} - \kappa \mathbf{\Delta t}) x_n + \sigma \sqrt{\mathbf{\Delta t}}\, \eta_n$$

- $\kappa$ linear decay rate per $\Delta t$
- normal form parameter $\alpha = \frac{\kappa^2}{4}$
- scaling $\frac{\text{avg slope of } x_{\text{trend}}}{\text{avg slope of } \kappa}$

# Estimate from time series — I



# Estimate from time series — II




**time series**
(ice-core record)

**fold ($a = 0$)**
**escape (early)**

# Summary — 2nd part

- noise causes early escape near tipping points
- probability depends on type of bifurcation and normal form parameters only
  ⇒study noise effects in normal form

- propensity to escape early can be estimated from time series
- strong non-normality of predicted tipping & escape times

⇒ **[JMTT, JS]** on arxiv