Tipping in a cellular automaton modelling forest-fire feedback in tropical forest

Bert Wuyts & Jan Sieber
University of Exeter

PNAS 120 (45), e2211853120, 2023
e-mail: b.wuyts@ex.ac.uk
twitter: @b3wu8

University of Exeter

EPSRC
Engineering and Physical Sciences Research Council

TiPES

OUTLINE

1. Stochastic cellular automaton of forest and fire

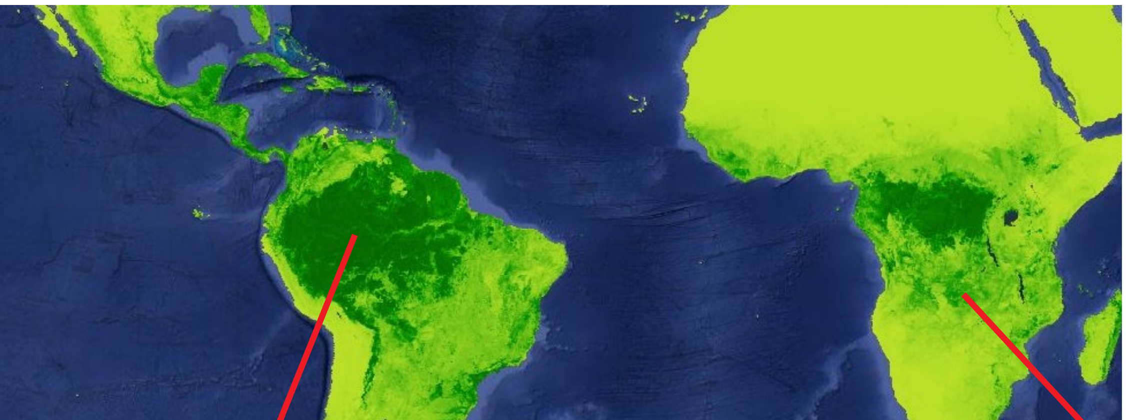
2. Slow and fast processes

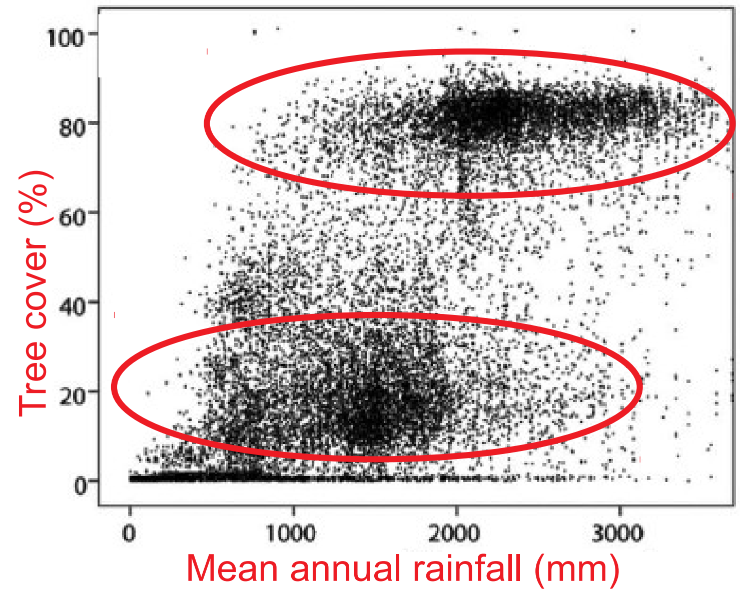
3. Analysis: steady states & dynamics

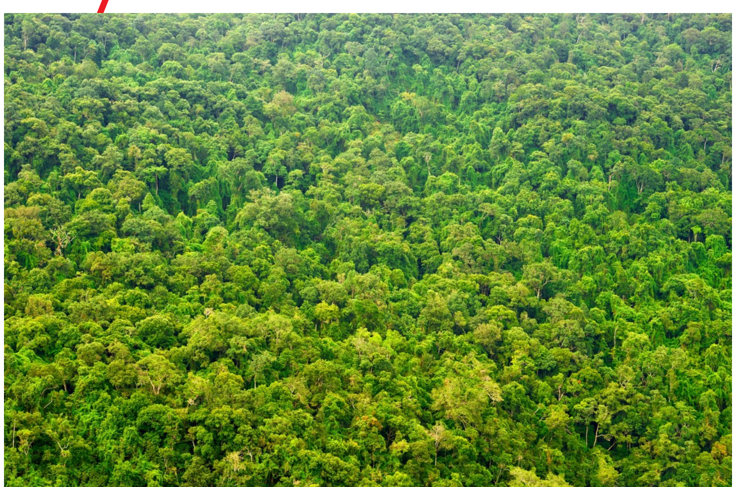
   => emergent structure-dynamics relations

Tropical tree cover bimodality

MODIS VCF data







Staver et al., 2011
Hirota et al., 2011

Stochastic cellular automaton:

fire and forest dynamics

CA FOREST AND FIRE

Square Lattice (each cell ~ 30m x 30m) $N \times N \quad (N = 100)$

4 Species: **T**ree, **G**rass, **B**urning, **A**sh

Empirical facts:

- fire ignites and spreads in grassland
- trees block fires but get damaged
- **fast** fire spread (hours-days)
- **slow** tree spread (years-decades)

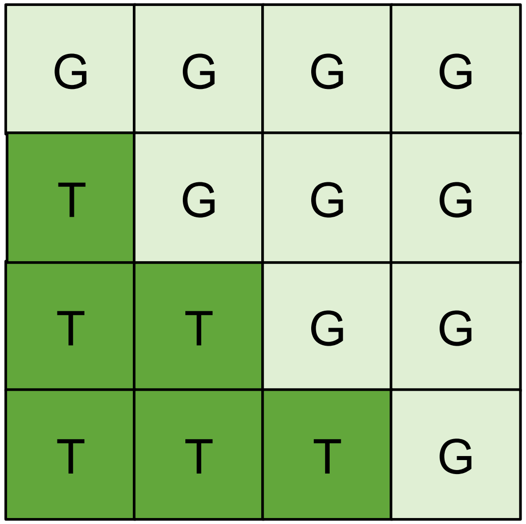
Staver et al., 2011; 2012
Dufresne et al., 2018

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



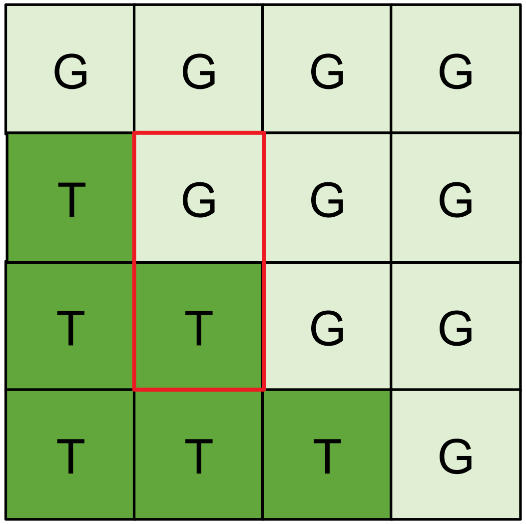
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	

G	G	G	G
T	T	G	G
T	T	G	G
T	T	T	G

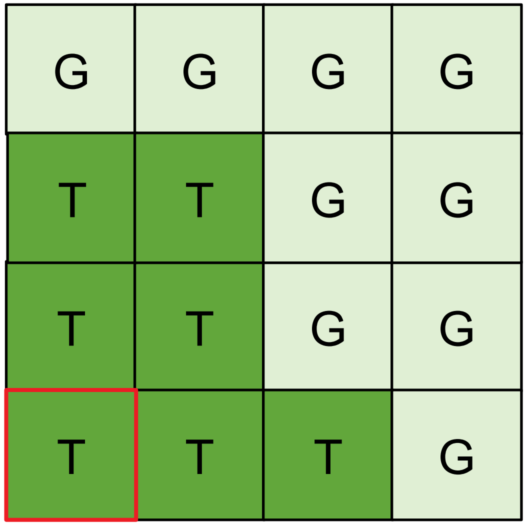
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	

G	G	G	G
T	T	G	G
T	T	G	G
G	T	T	G

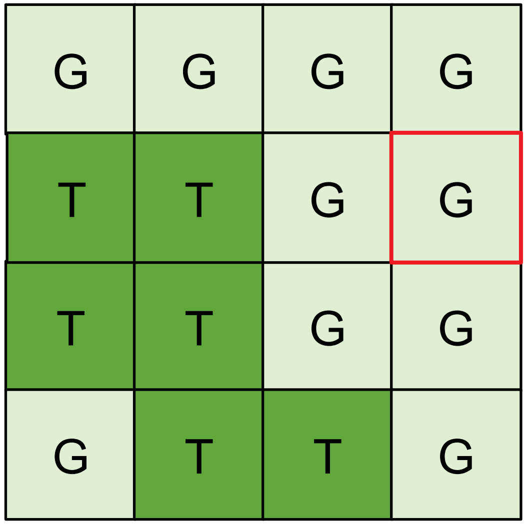
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	

G	G	G	G
T	T	G	T
T	T	G	G
G	T	T	G

Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	

G	G	G	G
T	T	G	T
T	T	G	G
G	T	T	G

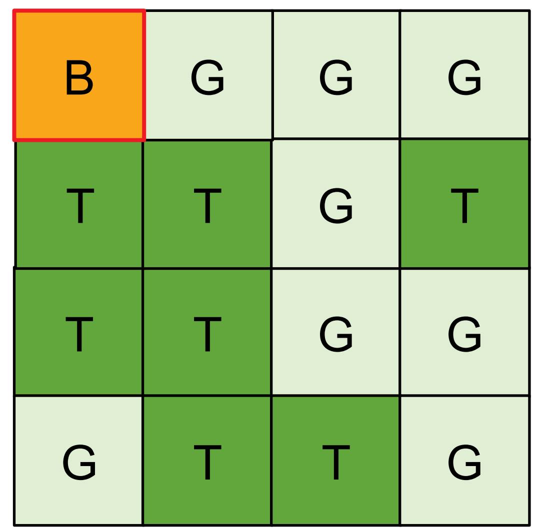
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



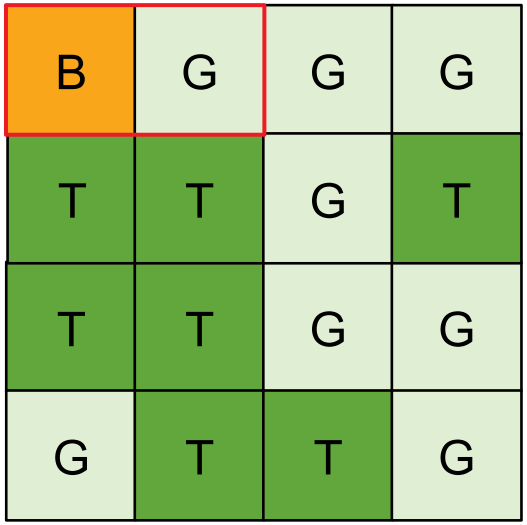
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	

B	B	G	G
T	T	G	T
T	T	G	G
G	T	T	G

Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	

B	B	B	B
T	T	B	T
T	T	B	B
G	T	T	G

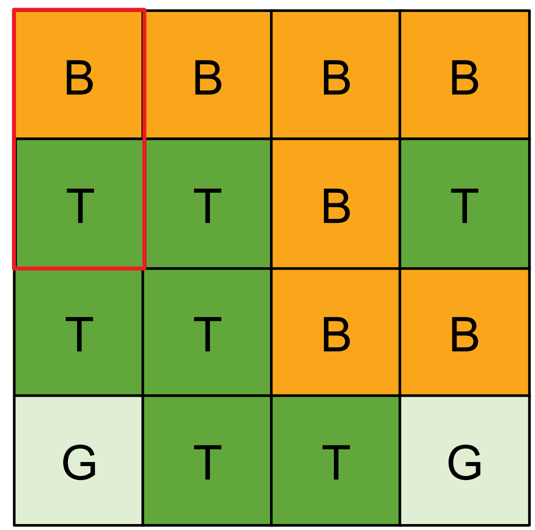
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



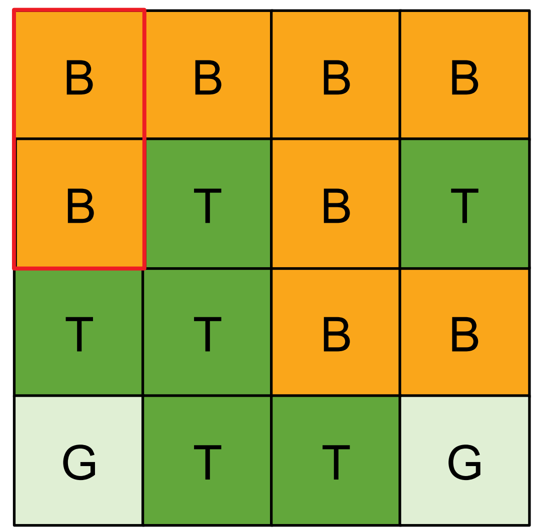
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



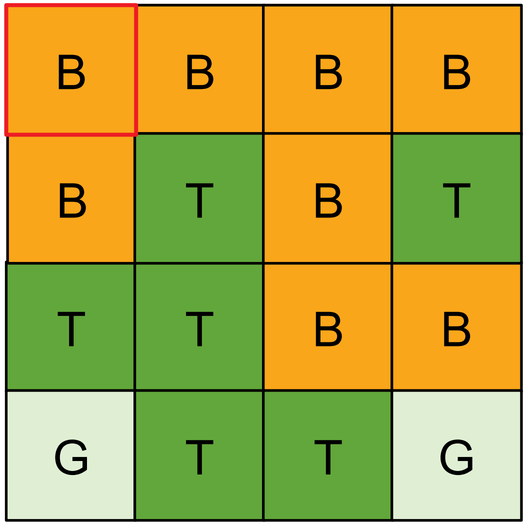
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



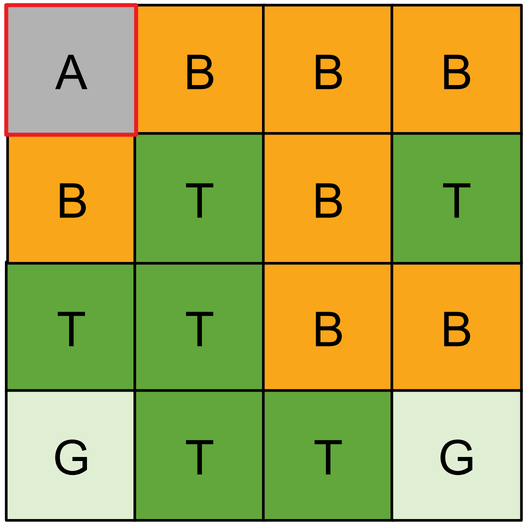
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



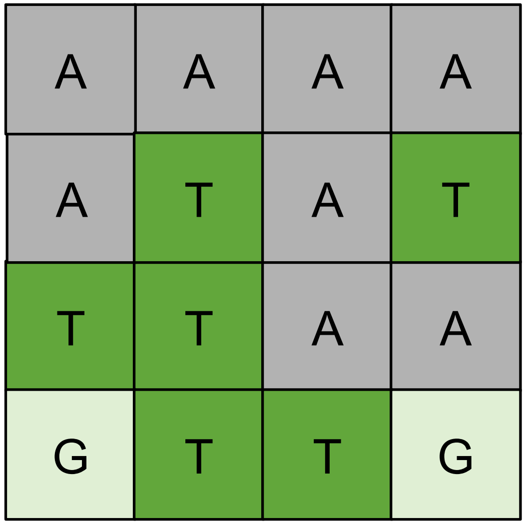
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



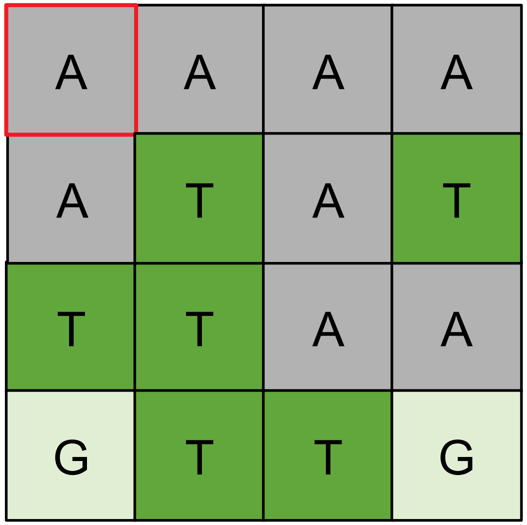
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



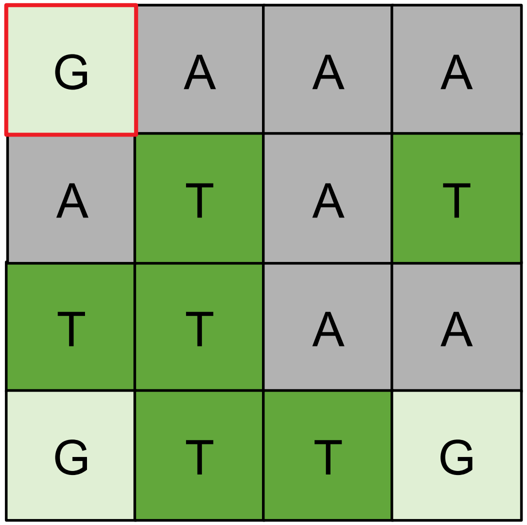
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



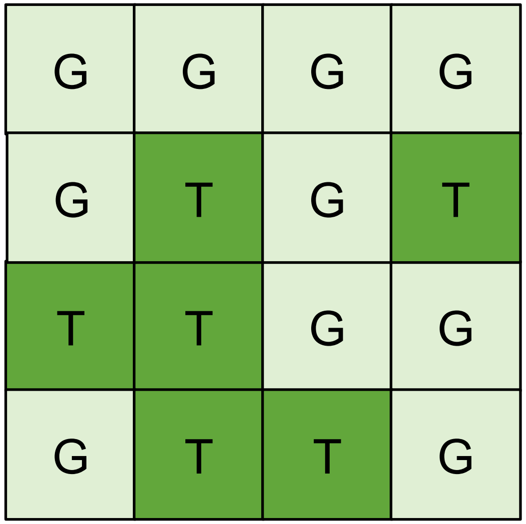
Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE

Cellular automaton:

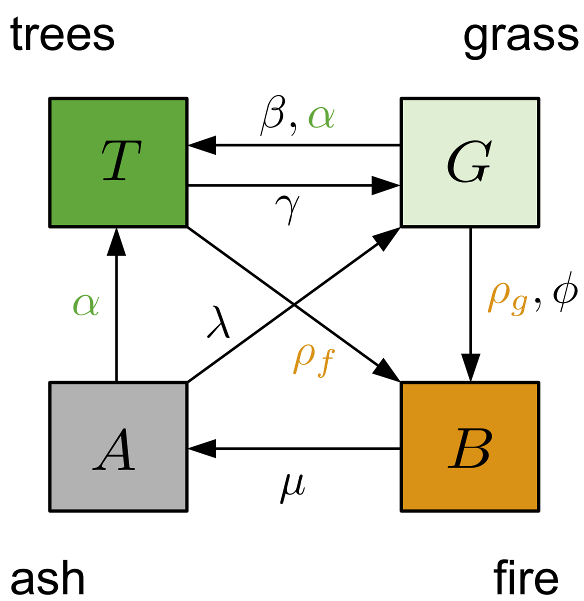
- species: {G, T, B, A}
- reactions:

Spontaneous	Spread
$G \xrightarrow{\beta} T$	$GT \xrightarrow{\alpha} TT$
$T \xrightarrow{\gamma} G$	$AT \xrightarrow{\alpha} TT$
$G \xrightarrow{\phi} B$	$GB \xrightarrow{\rho_G} BB$
$B \xrightarrow{\mu} A$	$TB \xrightarrow{\rho_T} BB$
$A \xrightarrow{\lambda} G$	



Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE



$$\mu, \rho_G > \rho_T \gg \lambda \gg 1 \gg \alpha, \beta, \gamma$$

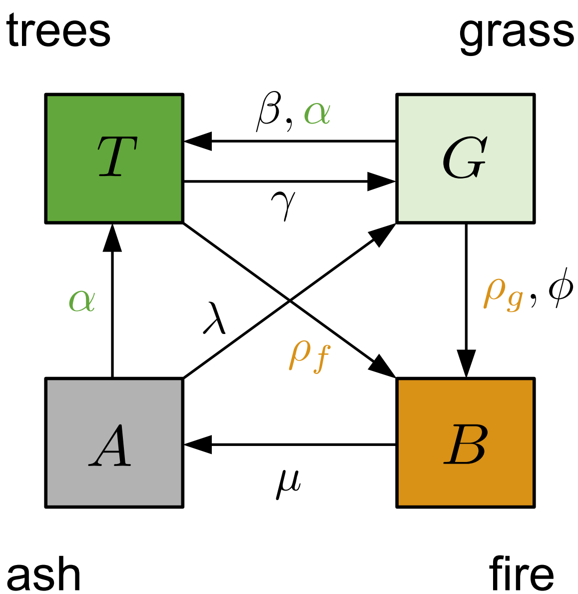
fast fire spread (<hours)

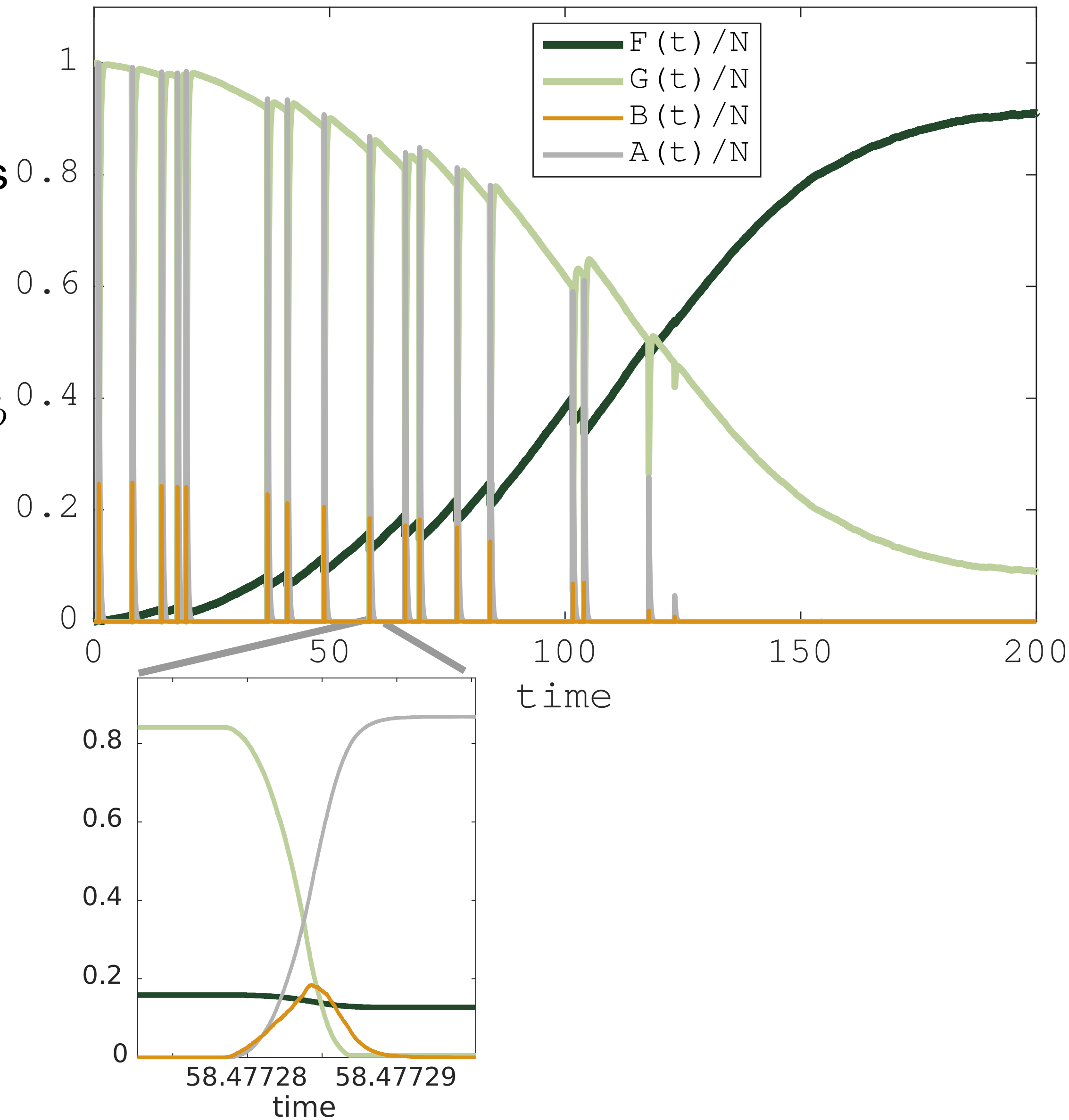
medium grass regrowth (months)

slow tree dynamics (decades)

Dufresne et al., 2018
Wuyts & Sieber, 2023

CA FOREST AND FIRE





video

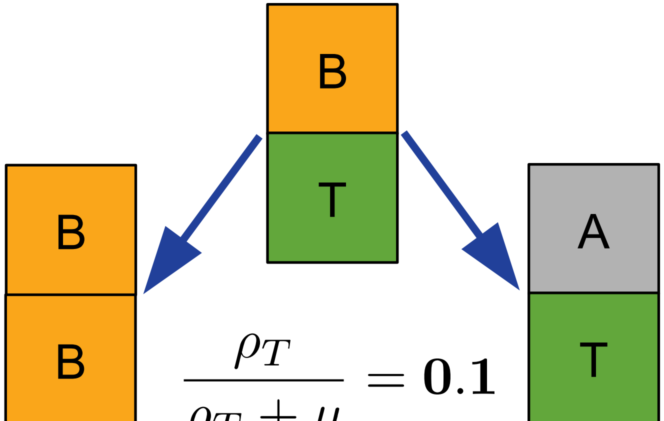
Dufresne et al., 2018
Wuyts & Sieber, 2023

Analysis of the forest & fire automaton

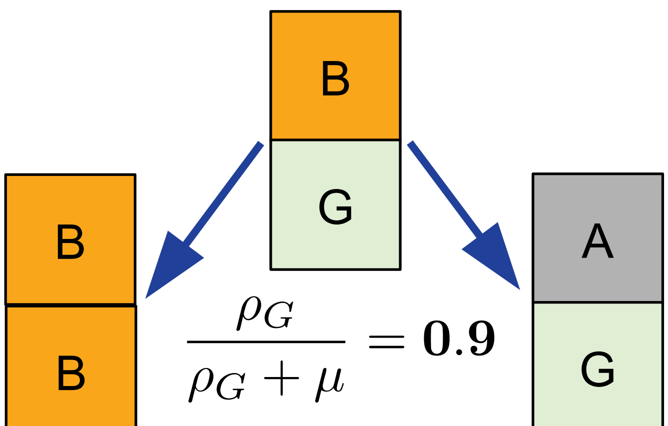
steady states,
dynamics
and
structure-dynamics relations

CA FOREST AND FIRE

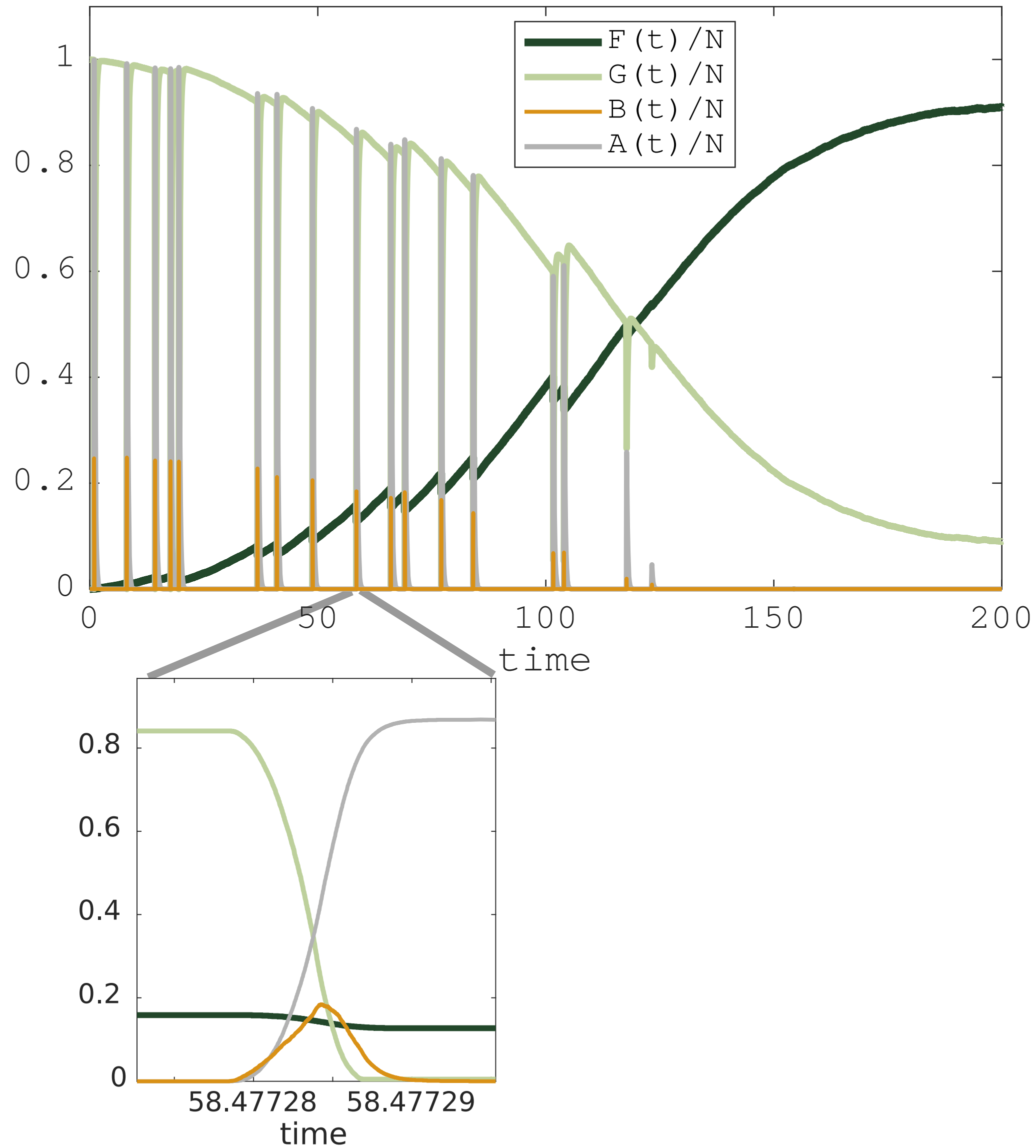
Fire spreading probability



$$\frac{\rho_T}{\rho_T + \mu} = \mathbf{0.1}$$



$$\frac{\rho_G}{\rho_G + \mu} = \mathbf{0.9}$$



Dufresne et al., 2018
Wuyts & Sieber, 2023

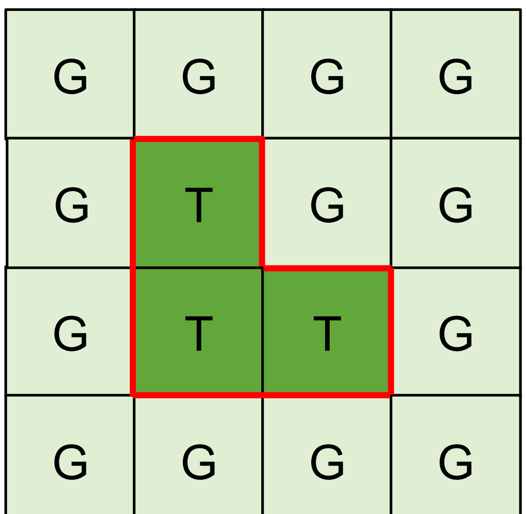
MACROSCOPIC QUANTITIES

Steady states and dynamics

=> first define macroscopic quantities:

- frequency of T or G cells: **[T], [G] (FOREST/GRASS AREA)**
- frequency of TG pairs: **[TG] (FOREST PERIMETER)**

e.g.:



normalised by N:

[T] = 3/16
[G] = 13/16
[TG] = 8/16

NOTE: mean field assumes
[TG] = 4 [T][G]

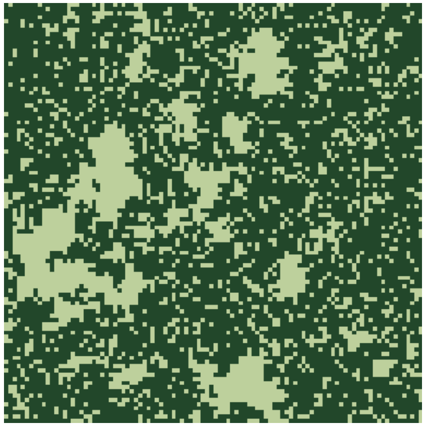
but:
[TG] = 0.5 **<** 4 [T][G] = 0.61

SLOW-FAST DYNAMICS

Fast dynamics

- For each grass patch, fires ignite and spread through entire patch to bounding forest:

  forest exposure to fire damage: $$\phi N \sum_{j=1}^{n_c} [G]_j [TG]_j$$



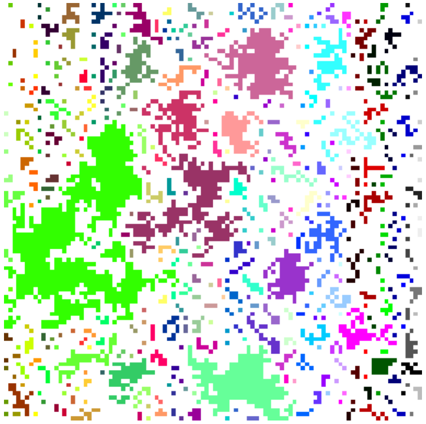
Slow dynamics

- fire-induced loss:

$$\Delta_T^{\text{loss}} := \sum_{j=1}^{n_c} \phi N [G]_j \times \frac{\rho_T}{\rho_T + \mu} [TG]_j$$

\# fires per time in cluster *j*

loss per fire in cluster *j*



- forest spread and spontaneous conversion:

$$\Delta_T^{\text{gain}} := \beta[G] - \gamma[T] + \alpha[TG]$$

Wuyts & Sieber, 2023

BALANCE ON SLOW TIMESCALE

$$q = \frac{\rho_T}{\rho_T + \mu} \approx 0.1$$

$$\frac{\mathrm{d}[T]}{\mathrm{d}t} = \beta[G] - \gamma[T] + \alpha[\underline{TG}] - \phi N q [G] \langle [\underline{TG}] \rangle_{\mathrm{cg}}$$

Overall
rate of change

forest perimeter
for spread

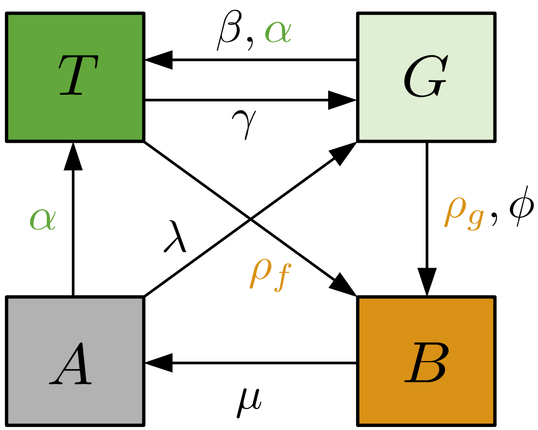
“**grassland-weighted**
forest perimeter”
for loss

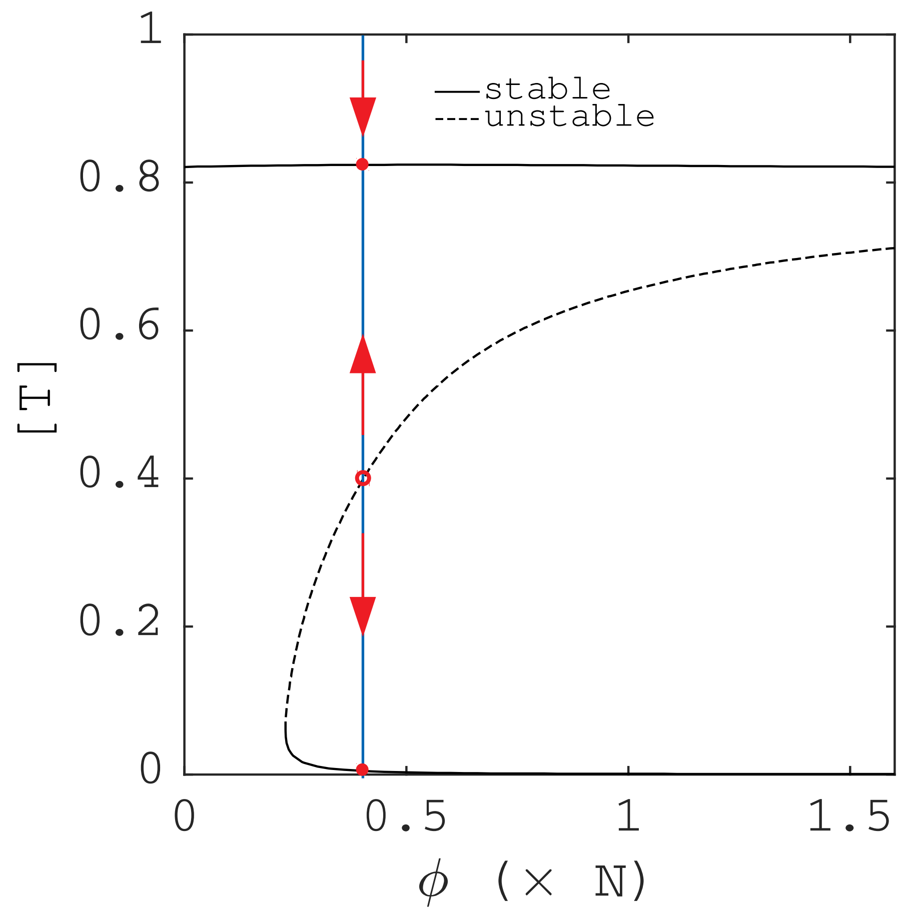
undetermined

$$\Downarrow \quad \langle [TG] \rangle_{\mathrm{cg}} := \sum_{j=1}^{n_c} \frac{[G]_j}{[G]} [TG]_j$$

Wuyts & Sieber, 2023

STEADY STATES VIA FEEDBACK CONTROL

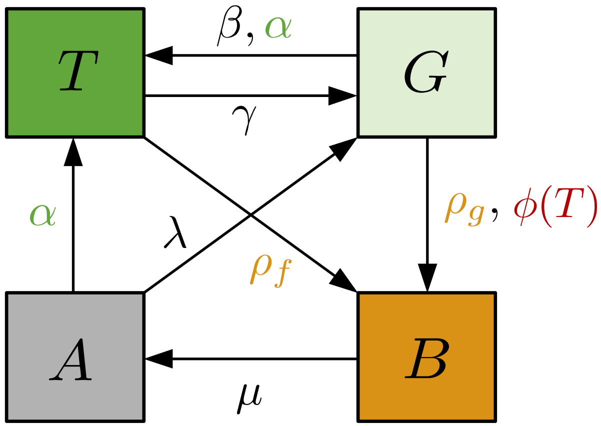


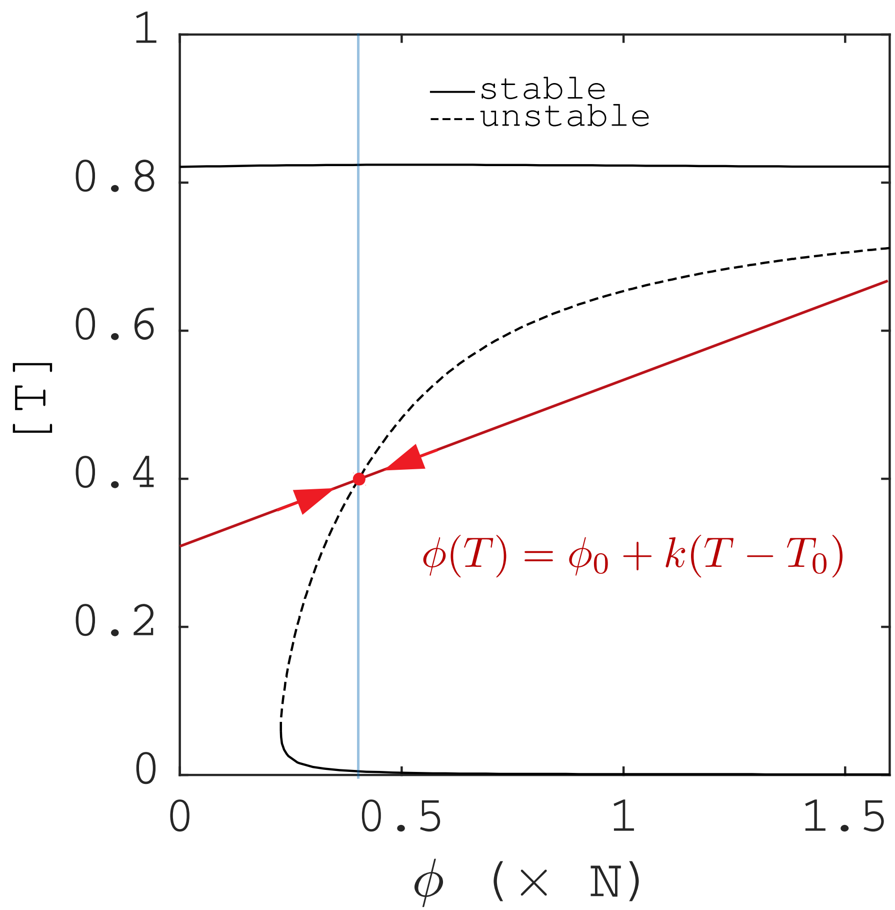


conventional simulation

Wuyts & Sieber, 2023

STEADY STATES VIA FEEDBACK CONTROL



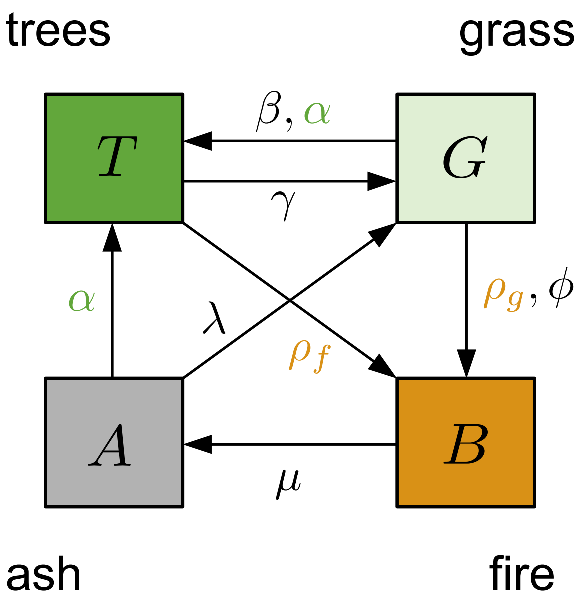


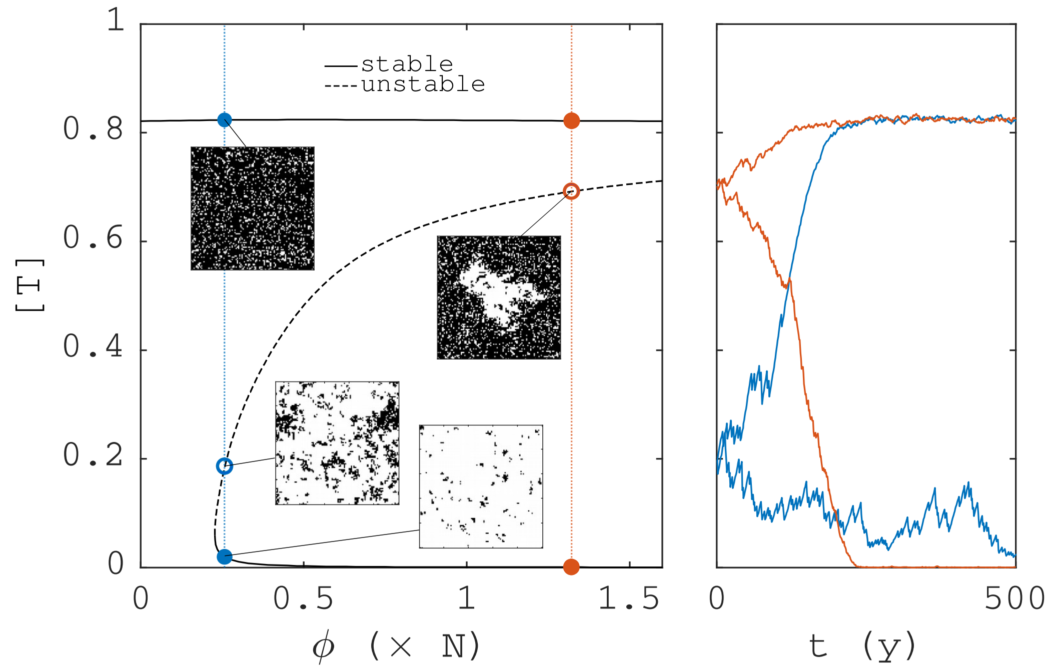
video (T0 = 40%)

simulation with feedback control

Wuyts & Sieber, 2023

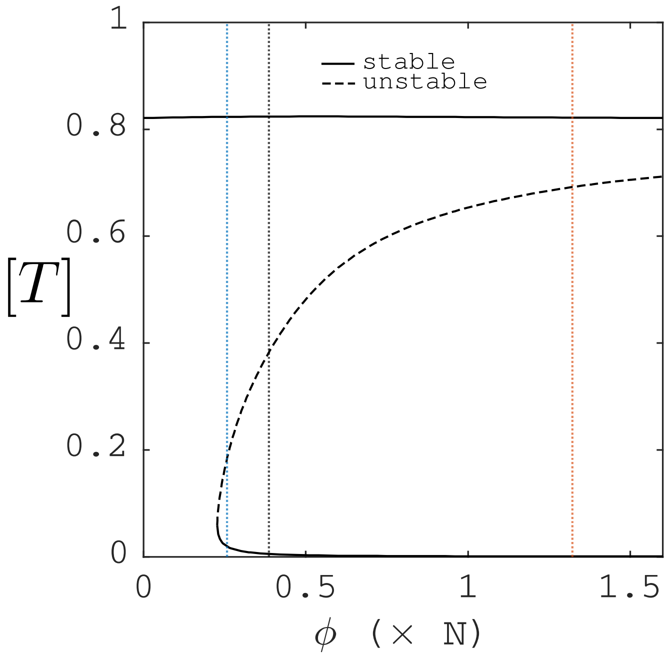
STEADY STATES & BISTABILITY





Wuyts & Sieber, 2023

CLOSURE



$$\frac{d[T]}{dt} = \beta[G] - \gamma[T] + \alpha[TG] - \phi N q[G]\langle[TG]\rangle_{cg}$$

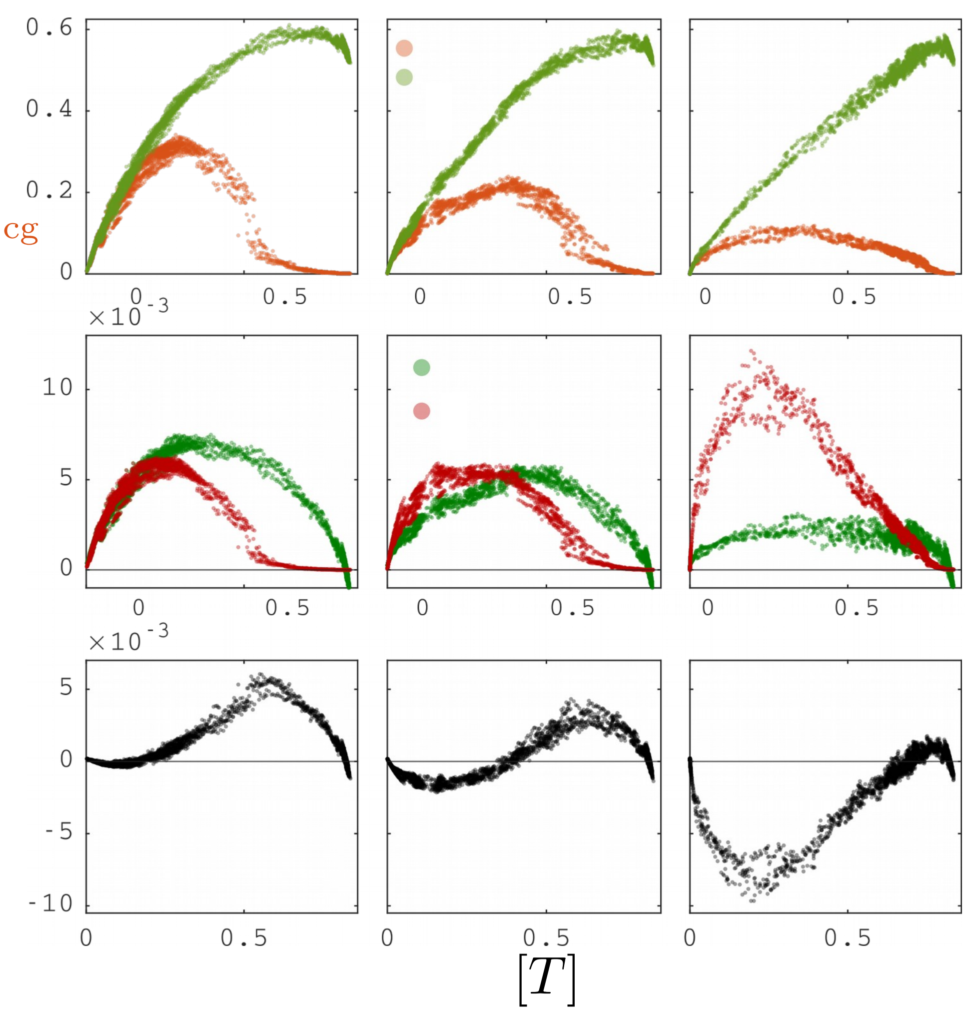
$[TG]$

$\langle[TG]\rangle_{cg}$

gain

loss

R.H.S. of emerging scalar ODE $\frac{d[T]}{dt}$



Wuyts & Sieber, 2023

STRUCTURE-DYNAMICS RELATIONS

$$\frac{\mathrm{d}[T]}{\mathrm{d}t} = \beta[G] - \gamma[T] + \alpha[TG]^* - \phi N q[G]\langle[TG]\rangle^*_{\mathrm{cg}}$$

linear terms (independent of spatial structure)

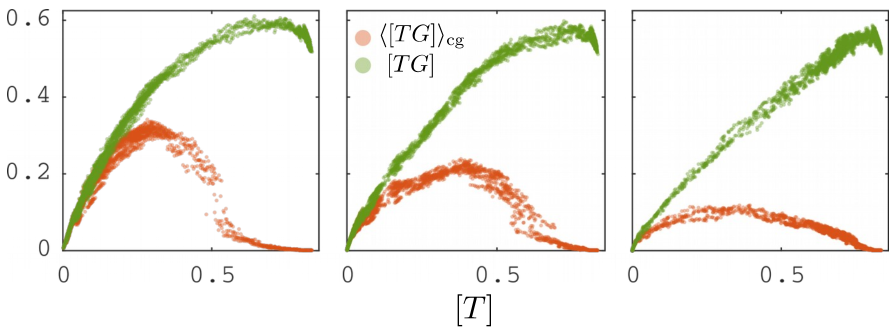
nonlinear terms (emerge from interactions at forest perimeter)

$[TG]^*([T])$: **perimeter-area** relation

$\langle[TG]\rangle^*_{\mathrm{cg}}([T])$: **weighted perimeter-area** relation

Can be calculated from spatial data

=> test where fire feedbacks are strong enough to cause bistability

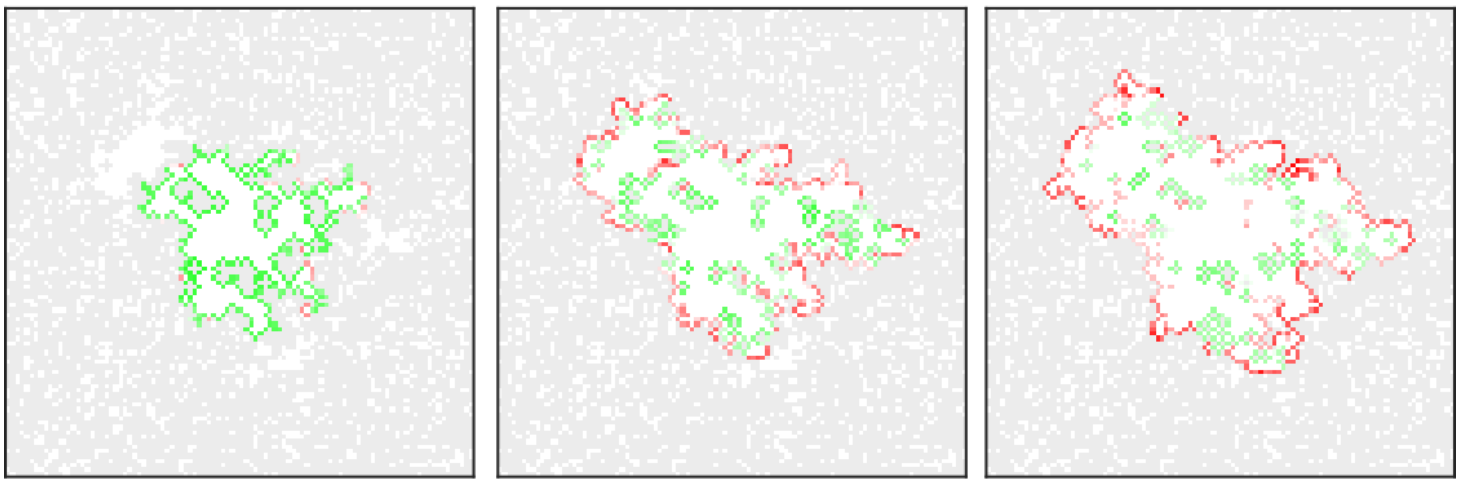


Wuyts & Sieber, 2023

EARLY-WARNING SIGNS

Hole of critical size in the forest

green boundary added	critical	red boundary removed



$\Delta[F]>0 \;\Rightarrow\; \frac{d}{dt}[F]>0$	$\frac{d}{dt}[F]=0$	$\Delta[F]<0 \;\Rightarrow\; \frac{d}{dt}[F]<0$

Wuyts & Sieber, 2023

CONCLUSIONS

- Hypothesis of tropical tree cover **bistability** relies on **bimodality** in observations, but alternative explanations exist
  => **more specific indicators** are required

- Processes on extremely different timescales
  => slow-fast analysis gives forest balance equation

- **Spreading processes occur near the forest perimeter**
  => structure-dynamics relations emerge
  => test where fire-vegetation feedbacks cause bistability

- Explore **saddle landscapes**

- Explore **Ash blocking fire** spread for frequent fires